

Faster-than- c signals, special relativity, and causality

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Abstract

Motivated by the recent attention on superluminal phenomena, we investigate the compatibility between faster-than- c propagation and the fundamental principles of relativity and causality. We first argue that special relativity can easily accommodate — indeed, does not exclude — faster-than- c signalling at the kinematical level. As far as causality is concerned, it is impossible to make statements of general validity, without specifying at least some features of the tachyonic propagation. We thus focus on the Scharnhorst effect (faster-than- c photon propagation in the Casimir vacuum), which is perhaps the most plausible candidate for a physically sound realization of these phenomena. We demonstrate that in this case the faster-than- c aspects are “benign” and constrained in such a manner as to *not* automatically lead to causality violations.

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1 Introduction

Recently, there has been a renewed interest in issues related to superluminal¹ propagation. This activity is connected with the experimental realization of faster-than- c group velocities [1], and with the theoretical discovery of the Scharnhorst effect — faster-than- c photon propagation in the Casimir vacuum due to higher order QED corrections [2, 3, 4, 5]. Although these two strands of research deal with apparently similar subjects, their implications on fundamental physics are quite different. The resonance-induced superluminal group velocities currently of experimental interest were already known, theoretically, in the sixties [6]. In spite of their large value (about 300 times c) and the many claims made in the non-specialized press, they do not create any problem of principle, because the *signal* velocity² still has c as an upper bound [8, 9, 10, 11, 12].

In contrast, the Scharnhorst effect, which predicts that in a cavity with perfectly reflecting boundaries, photons can travel at a speed slightly larger than c , is an extremely tiny phenomenon, well below our capabilities for experimental verification. It is nevertheless of fundamental theoretical importance, in that here it seems to be the actual signal speed that it is modified. Indeed, though the original derivations of the effect [2] are carried out in the “soft photon” regime (wavelengths much larger than the electron Compton wavelength), there is an argument (based on the Kramers-Kronig dispersion relations) which strongly suggests that the actual signal speed is enhanced with respect to the ordinary speed of light [3]. In the present paper, we adopt the view that these calculations, taken together, actually imply that the Scharnhorst effect is physical, and not an artifact of some approximation: it appears that quantum-polarization can induce *true* superluminal velocities, albeit well outside the realm of present day experimental techniques.

Regarding the generality of the effect, we stress that similar results have been found for the propagation of photons in a gravitational field [14] (note that also these calculations are based on the “soft-photon” approximation and their extendibility to higher frequencies is still uncertain). A framework which allows to discuss both types of effects in a unified fashion has been recently developed in [15, 16]; see also [17].

The Scharnhorst effect, and its gravitational versions, are at first disturbing because they indicate a violation of Lorentz invariance for the electromagnetic field in vacuum, which is commonly taken as a paradigmatic example of a Lorentz-invariant system. Of course, in the general relativistic framework of [14] it is *local* Lorentz invariance [18] that is violated. Upon closer inspection, however, Lorentz invariance is not affected at a fundamental level. Between conducting plates, or in a background gravitational field, light does not propagate at the usual speed, simply because the boundaries (or the background) single out a preferred rest frame, which shows up in the property of the quantum vacuum of not being Lorentz-invariant. Thus, light behaves in a non-Lorentz-invariant way only because the ground state of the electromagnetic field is not Lorentz-invariant. The Euler–Heisenberg Lagrangian, from which the existence of the effect can be deduced, as well as

¹Hereafter, by “superluminal” we always mean “faster than light in unbounded empty space”, *i.e.*, with speed larger than $c = 2.99792458 \times 10^8$ m s⁻¹. This terminology may produce some weird-sounding sentences, such as “light travels at a superluminal speed in the Casimir vacuum”, but it cannot lead to any misunderstanding if interpreted in the strictly technical sense described above.

²Following common usage, we identify the “signal velocity” with the phase velocity at infinite frequencies, equivalent to the so-called “front velocity” [4]. See [7] for other possible definitions.

all the machinery of QED employed in its derivation, are still fully Lorentz-invariant. For this reason, one often speaks of a “soft breaking” of Lorentz invariance, in order to distinguish from a situation in which also the basic equations, and not just the ground state, are no longer Lorentz-invariant.

Of course, this soft breaking of Lorentz invariance has no fundamental influence on special relativity — no more than being inside a material medium has. For many purposes, the quantum vacuum can indeed be regarded as a medium — an “æther” [19]. The fact that the Scharnhorst effect appears to violate the principle of relativity can then be understood by considering that, between infinite parallel plates, the æther is no longer Lorentz-invariant, so its presence, in contrast to the situation in infinite space with no boundaries, *can* be detected. Nevertheless one can always imagine experiments based on interactions which are unaffected by the QED vacuum, such as gravity, that would show no violation of Lorentz invariance. Only if the deviations were universal, which is not the case, would special relativity be threatened (and even in that case, one could easily avoid troubles by adopting a metric different from the Minkowski one, *i.e.*, by shifting to a *general* relativistic context).

There is another unpleasant feature of Scharnhorst’s result, though. As we pointed out, any refractive medium leads to a soft breaking of Lorentz invariance, as shown by the fact that the speed of light differs from the value c . However, in all other known cases, the speed of light is *smaller* than c , while, according to Scharnhorst, between plates it should be *greater*. This is a conceptually nontrivial result, because it contradicts the common belief that signal propagation at speeds larger than the speed of light in vacuum is forbidden by special relativity. Furthermore, it shows that faster-than- c effects can occur even within well-established theories such as QED.

The main goal of the present paper is to show that faster-than- c propagation does not violate the basic tenets of special relativity, and is kinematically perfectly compatible with it (section 2). In addition, we shall prove in section 3 that the existence of phenomena such as the Scharnhorst effect does not necessarily lead to closed causal curves, contrary to some claims made in the literature [20]. In particular, for a single pair of Casimir plates photon propagation can be described by an “effective metric” that exhibits the property of “stable causality”, which automatically prevents the formation of closed timelike loops. When one considers multiple pairs of Casimir plates, naive attempts at producing causality violations are vitiated by their need for either grossly unphysical inter-penetrating plates (which undermine the whole basis for the Scharnhorst calculation), or by violent and uncontrolled edge effects. The general situation is best analyzed in terms of Hawking’s chronology protection conjecture.

Section 4 contains a brief summary of the main results of the paper. Although it does not challenge present-day physics, the possibility of faster-than- c signalling nevertheless brings out some interesting open issues about the foundations of special relativity. Since these issues, although related to some points touched in our discussion, are not directly relevant for the main topic of the paper, we present them in the three appendices.

2 Special relativity

There is a logical distinction between the concept of an *invariant* speed, and that of a *maximum* speed. Contrary to widespread belief, special relativity only requires, for its

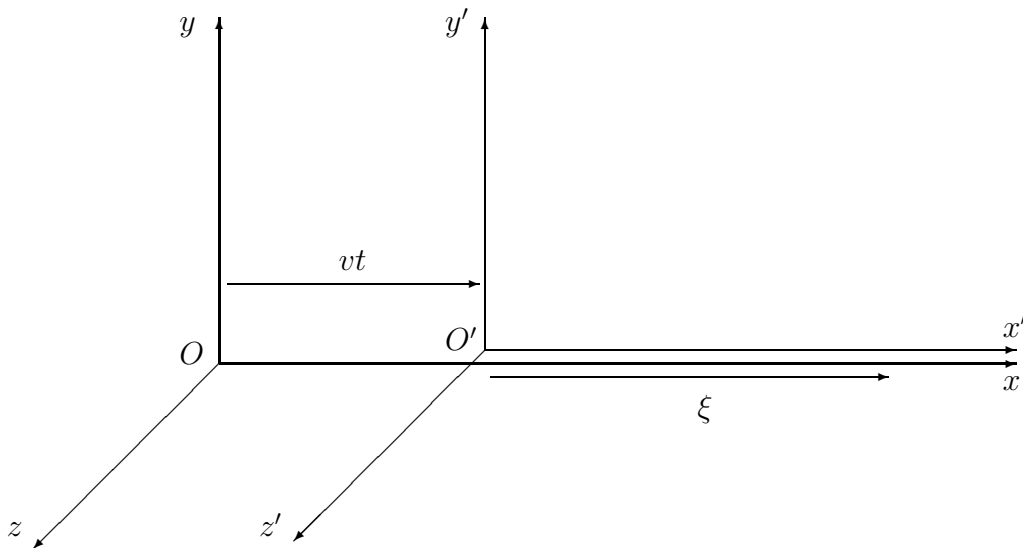


Figure 1: Spatial distances as measured in the reference frame \mathcal{K} ; see equation (2.1).

kinematical consistency, that there be an invariant speed. This is already obvious even from the original derivation by Einstein of the Lorentz transformations [21]. However, Einstein’s derivation relies upon a procedure of clock synchronization in which light signals are used, and one might think that if faster-than-light propagation is possible, it could be used to synchronize clocks in an alternative way, thus undermining relativistic kinematics at its very foundations. In this section we show that this is not the case.

2.1 Lorentz transformations without light

We find it convenient to start from an alternative, less well-known derivation of the Lorentz transformations, first discussed by von Ignatowsky in 1910 [22], and later rediscovered by many people [23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. The starting hypotheses are:

- (i) That in an inertial frame space is homogeneous, isotropic, and Euclidean, and time is homogeneous;
- (ii) The principle of relativity;
- (iii) A condition of “pre-causality” (see below).

These postulates alone imply the existence of the Lorentz group, containing *some* parameter $C > 0$ that represents an *invariant* speed. We now sketch the main lines of the argument. A rigorous and detailed presentation can be found in the papers by Gorini and collaborators [26, 27]; see also the book by Torretti [32] for a good summary, and [28, 29, 30] for pedagogical introductions.

Let us suppose that, in two inertial frames \mathcal{K} and \mathcal{K}' , with spatial origins O and O' , respectively, the same event is labelled by coordinates (t, x, y, z) and (t', x', y', z') . Because of isotropy of space, we can orient the spatial axes so that the relative velocity between

the frames is along x and x' . Since the relationship between the other coordinates turns out to be trivial, we shall ignore them, assuming that $y = y' = z = z' = 0$, and focus only upon (t, x) and (t', x') .

According to the reference frame \mathcal{K} , the spatial distance between the event and the origin O is obtained by summing the distance of the event from O' measured in \mathcal{K} , say ξ , to the distance between O and O' . The latter is simply vt , where v is the velocity of O' with respect to \mathcal{K} , so

$$x = \xi + vt \quad (2.1)$$

(see figure 1). Similarly, if we call ξ' the distance between the event and O according to \mathcal{K}' , and v' the velocity of O with respect to \mathcal{K}' , we have

$$x' = \xi' + v't' . \quad (2.2)$$

Now, homogeneity of space requires that the relationship between the distance of the event from O' , measured in the two frames, be linear. Thus, there exists a constant coefficient γ (possibly dependent on v and v') such that $\xi = x'/\gamma$. Similarly, there will be a γ' such that $\xi' = x/\gamma'$. Of course, γ and γ' must be positive, because they relate measurements of distances. Replacing these expressions into equations (2.1) and (2.2), and solving with respect to t' and x' , we get:

$$t' = -\frac{\gamma v}{v'} \left(t - \frac{\gamma\gamma' - 1}{\gamma\gamma'v} x \right) ; \quad (2.3)$$

$$x' = \gamma(x - vt) . \quad (2.4)$$

Until now, we have only used the hypothesis of homogeneity and isotropy of space. A second ingredient in the derivation is the so-called *reciprocity principle*, which asserts that $v' = -v$. (This is a non-trivial consequence of the isotropy of space [26, 29, 32].) In addition, the principle of relativity imposes the further constraint that $\gamma' = \gamma$, and it requires that the transformations form a group. From the latter condition one can then easily show, by considering three inertial frames, that $\gamma^{-2} = 1 - kv^2$, with k a v -independent constant.

The case of a negative k can be excluded by the following argument. If $k < 0$, on defining $\tau := t/\sqrt{-k}$ and $\tan^2 \psi := -kv^2$, the resulting transformation is easily recognized to be an ordinary rotation of an angle ψ in the (τ, x) plane. This has a number of unphysical consequences. In particular, let us consider two events with the same x coordinate in \mathcal{K} , and different t coordinates; then it is always possible to change reference frame in such a way that the time order of the events is reversed.³ This result would clearly prevent the application of any meaningful notion of causality. In order to avoid this possibility, we impose the following “pre-causality” requirement, which is a necessary condition for causality to be defined: *If two events happen at the same place in one reference frame, their time order must be the same in all reference frames.*⁴ This implies $k \geq 0$, so we can define a (possibly infinite) positive constant C such that $k = 1/C^2$. The transformation (2.3)–(2.4) thus takes on the familiar Lorentz form, with the speed of light replaced by the

³This is most easily seen by applying two successive transformations with “speeds” v_1 and v_2 , say, such that $v_1 v_2 > -1/k$.

⁴Equivalently, one might ask that the composition of two velocities in the same direction does not produce a velocity pointing in the opposite direction [29, 30].

constant C . It is then obvious, considering the associated composition law for velocities, that if a signal travels at the speed C in one reference frame, it does so in all reference frames. That is, C is an invariant speed.

Note that nowhere in the previous argument have we assumed the existence of a maximum speed. The principle of relativity, together with the requirements of homogeneity and isotropy, is sufficient in order to obtain a Lorentz-like transformation. Nor have we deduced that signals cannot propagate at a speed greater than C . We have only found that one cannot consider *reference frames* in motion with relative speed $v > C$ (otherwise γ would turn out to be imaginary), but of course this does not mean that there is an upper bound to signal velocity. Even if it were possible to send signals at arbitrarily high speeds, the kinematical group could still have the Lorentz form with a finite value of C . This is a counterintuitive result, because one might think that if arbitrarily fast signalling were possible, then one should be able to synchronize clocks in an absolute way.

2.2 Fluid-dynamical analog

Let us try to clarify this point by using an analogy. Consider a fluid at rest with respect to an inertial frame, within the context of *Newtonian* mechanics, and two reference frames, one coinciding with the rest frame of the fluid, the other moving with respect to it with a constant velocity \mathbf{v} . Furthermore, let us assume that the axes of the two frames are parallel, and that \mathbf{v} is directed along one of them, so we don't need to care about the other two. If the observers in the two frames agree to use ultra-fast signals (certainly allowed by Newtonian physics) in order to synchronize their clocks, the relationship between the coordinates (T, X) of the frame at rest with respect to the fluid, and the coordinates (t_G, x_G) of the other frame is the usual Galilean transformation:

$$t_G = T ; \tag{2.5}$$

$$x_G = X - vT . \tag{2.6}$$

Using these coordinates, the propagation of waves in the fluid will be described by the two frames in a different way. For two events on the wavefront of a sound wave propagating along the X direction, separated by coordinate lapses ΔT and ΔX , we have

$$-c_s^2 \Delta T^2 + \Delta X^2 = 0 , \tag{2.7}$$

where c_s is the speed of sound in the frame at rest with respect to the fluid. On the other hand, in (t_G, x_G) coordinates we have

$$-c_s^2 \Delta t_G^2 + (\Delta x_G + v \Delta t_G)^2 = 0 , \tag{2.8}$$

where we have inserted the Galilean transformations (2.5) and (2.6) into equation (2.7). Obviously, equation (2.8) does not have the same form as equation (2.7), even replacing c_s by some new function of c_s and v . Thus, although the Galilean transformation does satisfy the principle of relativity, the description of sound propagation given by two sets of coordinates linked by a Galilean transformation is not symmetric. (Of course, there is no reason why it should be, because the presence of the fluid causes a soft breaking of Galilean invariance, since it defines an “absolute” rest frame.)

However, the moving observer could decide to *define* his coordinates in such a way that the principle of relativity is still satisfied *and* that the invariant speed is no longer

$C = +\infty$, as in the Galilean transformation, but $C = c_s$. To do this, it is sufficient that he use sound signals in order to synchronize his clocks, and to define distances by an acoustic “radar procedure”, as bats would do (this is possible if $v < c_s$). Calling (t_L, x_L) the coordinates defined in this way, we have

$$t_L = \gamma_s \left(T - vX/c_s^2 \right) , \quad (2.9)$$

$$x_L = \gamma_s (X - vT) , \quad (2.10)$$

where $\gamma_s = (1 - v^2/c_s^2)^{-1/2}$. Thus, the coordinates (t_L, x_L) are related to (T, X) by a Lorentz transformation with invariant speed c_s . It is obvious that the relationship between the lapses Δt_L and Δx_L corresponding to two events on a sound wavefront is now

$$-c_s^2 \Delta t_L^2 + \Delta x_L^2 = 0 , \quad (2.11)$$

i.e., it has exactly the same form as equation (2.7).⁵

This example brings out a fundamental issue: Given a propagation phenomenon that is associated with an equation like (2.7) in one reference frame, one can *always* find time and space coordinates, in any other frame, such that the principle of relativity is satisfied and an analogous equation holds true in the new frame. Thus, the Lorentz transformations corresponding to an arbitrary invariant speed C are, in general, devoid of physical meaning. One can always *choose* the value of C , because such a choice is essentially equivalent to giving a prescription for synchronization, selecting the class of signals used in order to synchronize clocks.⁶ One is then led to ask: Is not the situation just the same with the Lorentz transformations? If so, is the invariance of the speed of light just a convention, with no physical content?

2.3 Physical coordinates

The missing link is given by the *interpretation* of the time and space coordinates. In general, one establishes a reference frame first by choosing physical units of time and distance, based on some physical phenomenon. Then, a Cartesian lattice is constructed using the standards of length, and clocks are (ideally) placed at all points in space. All these clocks tick at the same rate, but they are not yet synchronized. However, it is already possible to decide empirically whether some signals have an invariant speed. It is sufficient that an observer sitting at a point P , say, send a signal to another observer at a point Q of the same frame, and that this one immediately signal back to P . On measuring the time taken by the round trip on his clock, and knowing his distance from Q , the observer at P can readily establish the average value of the signal speed.⁷ If,

⁵As a side remark about this analog, notice that one could introduce a matrix with coefficients $G_{\mu\nu}$ such that $G_{\mu\nu} \Delta x^\mu \Delta x^\nu = -c_s^2 \Delta T^2 + \Delta X^2 = 0$ in *any* system of coordinates x^μ . One then gets, in Galilean coordinates,

$$[G_{\mu\nu}] = \begin{bmatrix} -(c_s^2 - v^2) & v \\ v & 1 \end{bmatrix} , \quad (2.12)$$

which is just the so-called acoustic metric, describing the propagation of perturbations in a fluid that moves with velocity $-\mathbf{v}$ [33].

⁶In this respect, note that in equations (2.9) and (2.10) the factor γ_s re-defines units, while the term vX/c_s^2 corresponds to a choice of synchronization.

⁷Note that the one-way speed cannot be measured unless clocks at different places are synchronized [31, 34, 35, 36, 37, 38, 39].

repeating this kind of measurement along different directions, and in different frames equipped with the same type of clocks and rods, one always obtains the same value, these signals travel at the invariant speed. In the Newtonian fluid dynamic example of the previous subsection, this speed is infinite. It is important to emphasize that, since units of time and distance have been chosen in advance, the value of the invariant speed cannot be chosen at one's will. Hence, the existence of an invariant speed of some specific value, albeit a two-way average speed, is a nontrivial physical law, and not a consequence of an arbitrary convention (see Reichenbach [34], pp. 204–205).

The next step is the prescription for the synchronization of clocks at different places — hence, for the definition of simultaneous events in a given reference frame. This *is* a conventional element in the theory [31, 34, 35, 36, 38, 39] (however, see [37] for a different opinion). It is clearly embodied in the postulates (i) and (ii), which imply a well-defined relationship between the coordinates in two different inertial frames, and hence a well-defined notion of which events are simultaneous in an arbitrary frame. This, in turn, is equivalent to postulating that the speed C be invariant not only on average during a round trip, but also in the one-way sense.

Of course, one could use the signals travelling at the invariant speed in order to synchronize clocks, but we want to understand what would happen using other signals. It is easy to see, however, that if the coordinates in the two reference frames are related by a Lorentz transformation with invariant speed C , synchronization *must* be performed using signals that travel at the speed C . In order to prove this, let us consider again the framework described in section 2.1 (see figure 1), and let us suppose that both reference frames agree to use signals that travel at some given speed V in \mathcal{K} . Then, if an observer at O' sends two such signals along the axis x' in opposite directions, they will reach observers in \mathcal{K}' with x' coordinate equal to l and $-l$, say, at the times

$$t'_+ = l \frac{1 - Vv/C^2}{V - v} \quad (2.13)$$

and

$$t'_- = l \frac{1 + Vv/C^2}{V + v}, \quad (2.14)$$

respectively. (Here, we have applied the law of composition of velocities that follows by the assumption that the coordinates in \mathcal{K} and \mathcal{K}' are related by a Lorentz transformation with invariant speed C .) If signals travelling at speed V have been used to synchronize clocks, then we must have $t'_+ = t'_-$, which is possible only if $V = C$.

Now there are two possibilities. If the value of the invariant speed has already been established experimentally, using the method outlined above, this result forces us to use precisely the invariant signals, and not others, in order to synchronize clocks. Alternatively, we might want V to be the invariant speed, as in the example of the fluid, where we defined (t_L, x_L) in such a way that c_s turns out to be invariant. Of course this is possible, but it requires the use of time and space coordinates that behave differently from those associated with the physical clocks and rods.⁸ In other words, while those

⁸This can be codified into a somewhat unusual “take” on special relativity: The Maxwell equations, considered simply as a mathematical system, possess a symmetry, the Lorentz group, under redefinitions of the labels x and t . But this is a purely mathematical statement devoid of interesting physical consequences until one asks how physical clocks and rulers are constructed, and what forces hold them together. Since it is electromagnetic forces balanced against quantum physics which holds the internal structure of these

Lorentz transformations associated with the physical invariant speed measured using independent standards of time and length connect actual measurements performed in two different frames, all other logically possible Lorentz transformations must be regarded as mere changes in the labels for the events, with no physical meaning. For example, in our fluid analog, the physical measurements of time and space intervals are ΔT and ΔX in all reference frames, according to the postulates of Newtonian mechanics. This selects automatically the transformation with $C = +\infty$ as the physically relevant one, among all those allowed by the relativity principle alone.

However, it is interesting to notice that if the observers were restricted to using the fluid in their measurements, *and nothing else*, the “physical” coordinates would probably turn out to be t_L and x_L . Indeed, hypothetical beings whose internal structure was completely mediated by phonon exchange, and whose rulers and clocks were likewise held together by phonon exchange, and who were completely blind to electromagnetism (so in particular they could not probe the atomic structure of individual atoms) would discover an “acoustic Lorentz invariance” with their “rulers” and “clocks” transforming according to the laws of an approximate “acoustic relativity” (the Lorentz group with $C = c_s$).⁹ In this case one might say that the measuring devices are dynamically affected by the motion with respect to the fluid, in such a way that such motion becomes undetectable. In a sense, the fluid would play the role of an æther whose presence is masked by length contraction and time dilation effects. Thus, we would have a fluid dynamical analog of special relativity *à la* Lorentz [40].¹⁰

2.4 What is C ?

Let us summarize. Homogeneity and isotropy of space in an inertial frame, together with the principles of relativity and causality, imply the kinematical Lorentz group, hence the existence of some invariant speed C . Clocks at a distance must be synchronized using signals that propagate at the speed C . The value of C is fixed once a prescription for constructing clocks and rulers is given and adopted uniformly in all reference frames.

Since the argument that establishes the existence of an invariant speed C is entirely of a kinematical character, it does not allow one to know the value of C . In principle, one could even have the degenerate case $C = +\infty$, which corresponds to Galilean relativity.¹¹

objects together, the experimental observation that to very high accuracy physical bodies also exhibit Lorentz symmetry allows one to deduce that quantum physics obeys the same symmetry as the Maxwell equations. Viewed in this way, all experimental tests of special relativity are really precision experimental tests of the symmetry group of quantum physics.

⁹Of course for normal observers, made of normal condensed matter, physical rods and clocks are seen (to good approximation) to transform according to the $C = c$ Lorentz group; normal observers would not see this “acoustic relativity”.

¹⁰Note that such an analogy cannot be strictly true in the case of the fluid. As a matter of fact, measurements performed by our hypothetical “fluid observers” would be insensitive to non-hydrodynamical physics only if they were probing a regime where the symmetry group of quantum physics (presumably the Lorentz group with $C = c$) did not overwhelm the acoustic effects. Moreover, even remaining in a classical regime, fluids definitely possess a minimum length scale (of the order of the intermolecular distance) under which the hydrodynamical approximation breaks down. This fact would by itself lead to a violation of Lorentz invariance at small scales and modifications of the dispersion relation for phonons at high frequencies. We shall come back to this point in our appendix A.

¹¹It has been argued, within the context of constructive axiomatic theories of spacetime structure, that signals with a maximum speed must exist, if one wishes to introduce coordinates by a radar technique [41].

To determine the actual value of C when a choice for clocks and rulers has been made is an experimental problem. We know that C coincides, to a very good accuracy, with the value c of the speed of light in a vacuum with no boundaries, so we shall set $C \equiv c$ in the rest of this paper. However, one should always keep in mind that tiny deviations from this empirically established equality are logically possible.

Once a finite value for C is found experimentally, the whole formalism of special relativity follows on the basis of postulates (i)–(iii). Since faster-than- C signals do not contradict any of these postulates, their possible existence is (kinematically) perfectly acceptable within ordinary special relativity. In particular, one could not make use of these signals in order to synchronize clocks without violating (i)–(ii) above. In fact, although different frames *could* use hypothetical ultra-fast signals in order to define the time coordinates t and t' in such a way that $\Delta t = 0$ whenever $\Delta t' = 0$, the use of such coordinates would be incompatible with the principle of relativity. Indeed, if $\Delta t = \Delta t' = 0$, equation (2.3) implies $\gamma\gamma' = 1$ and $t' = (-\gamma v/v')t$. Assuming isotropy of space gives the reciprocity relation $v' = -v$, so $t' = \gamma t$. If $\gamma \neq 1$ — an experimental issue that can be decided by comparing coordinate clocks in the two frames — this requires $\gamma' \neq \gamma$, in violation of the principle of relativity. The resulting transformation allows, of course, for an absolute notion of simultaneity, essentially due to the introduction of a preferred frame in the formalism [35, 39, 42]. Note that if $\gamma \neq 1$ it is impossible to set clocks so that $t = t'$, unless one accepts that reciprocity, hence isotropy of space, is also violated.

Hence, even if there were tachyons (*i.e.*, particles or signals faster than C), one could not use them in order to synchronize clocks in a frame-independent way, without contradicting the relativity principle. Of course, they could be used to define some t and x coordinates — just as *any* other signal (see [30], p. 30). But for synchronization one must use signals travelling at the invariant speed C , if one wants to satisfy the principle of relativity.

3 Causality

We have seen in the previous section that, once units of length and time are defined, purely kinematical arguments based on postulates (i)–(iii) lead to the existence of an invariant speed. Experimentally, the latter turns out to coincide with the speed of light c when we use “ordinary” clocks and rods. This, however, does not necessarily mean that nothing can propagate faster than c . As we have already discussed, the existence of signals faster than c would not threaten kinematic features of special relativity, because they could not be used to synchronize clocks in a physically acceptable way. Hence, a model of the world based on a four-dimensional spacetime with the Minkowski metric

$$\eta = -c^2 dt^2 + dx^2 + dy^2 + dz^2, \quad (3.1)$$

Of course, this indicates a difficulty of the radar technique in the case $C = +\infty$, rather than the logical impossibility of signals with infinite speed. Newtonian spacetime provides an example of a consistent spacetime model to which the axiomatic framework of reference [41] cannot be applied. Another problem for theories of this type arises precisely in the Casimir vacuum. Since the maximum speed is used in order to define the causal structure of spacetime, the existence of the Scharnhorst effect should lead one to adopt, inside plates, a spacetime metric different from the one outside. Thus, one would have a modification of the spacetime structure in a situation where the gravitational effects are certainly negligible.

in which particles can travel along timelike, null, and spacelike lines, is perfectly consistent from a kinematical point of view [24, 43]. In other words, special relativity can kinematically accommodate faster-than- c propagation.¹²

3.1 Tachyons and causal paradoxes

On the other hand, tachyons are usually associated with unpleasant causal paradoxes. The basic reason for this belief is that if two events, say \mathcal{E}_1 and \mathcal{E}_2 , are spacelike related, there is no absolute time ordering between them. Thus, if a signal travels from \mathcal{E}_1 to \mathcal{E}_2 in a reference frame, it is always possible to find another frame where \mathcal{E}_1 and \mathcal{E}_2 are simultaneous, so the signal would appear to travel at an infinite speed, and others where \mathcal{E}_1 happens after \mathcal{E}_2 , so the signal would “travel to the past”.¹³

Obviously, this argument could be used to criticize faster-than- c propagation only if, in any given reference frame, the only criterion for saying that an event is the cause of another one were the time ordering in that frame. However, cause and effect are usually *not* defined in this way.¹⁴ In fact, there are no precise definitions of these concepts, but only some intuitive ideas that allow us to recognize, in some cases, the existence of a causal relationship between events. Eventually, the criteria used to establish that \mathcal{E}_1 is a cause of \mathcal{E}_2 are based on considerations of complexity of the type usually involved in discussions about the so-called “arrow of time”. For example, if \mathcal{E}_1 represents the emission of a signal from a broadcasting station, and \mathcal{E}_2 its reception on a TV set, we consider \mathcal{E}_1 a cause of \mathcal{E}_2 not just because it happened at an earlier time, but mainly because the opposite choice would require the presence of weird, conspiratorial, correlations. Now, if \mathcal{E}_1 and \mathcal{E}_2 were connected by faster-than- c photons instead of ordinary ones, the same criterion would apply, so there would be an absolute notion of what is cause and what is effect. In this respect, the exchange in the time ordering of \mathcal{E}_1 and \mathcal{E}_2 within some reference frames

¹²It is nevertheless worth noticing that, although faster-than- c signals are not incompatible with special relativity, there is apparently a structural obstruction to forming stable tachyons [9].

¹³We are of course currently using the term “tachyon” in the sense of some postulated faster-than- c particle or signal. There is a subtly different usage of the word “tachyon” in a field-theoretic sense to refer to a field theory with canonical kinetic energy and negative mass-squared (imaginary proper mass). Despite the fact that the resulting dispersion relation mimics the mass-momentum relation of a faster-than- c particle, field-theoretic tachyons of this particular type cannot be used to send faster-than- c signals. Classically this arises because the characteristics of the field equations (the partial differential equations derived from the field theory) depend only on the kinetic energy term and are completely insensitive to the mass term. As long as the kinetic energy terms are canonical the characteristics are the usual ones, and the signal speed is c , regardless of whether the proper mass is positive, zero, or imaginary. This observation can be rephrased in terms of the evolution of initial data of compact support, for which it is a standard result of the theory of partial differential equations that canonical kinetic energy terms limit the support of the evolved field configuration to lie inside the light-cone of the initial data [9]. This result also holds quantum mechanically where canonical quantization requires equal-time commutators to vanish, which then extends via Lorentz invariance to vanishing of the field commutators everywhere outside the light cone; thus quantum mechanically the signal speed is also limited to c . For field-theoretic “tachyons” with canonical kinetic energies it is not causality that is the problem, rather it is the issue of the instability of the field-theory ground state that leads to difficulties. Note that for the Scharnhorst photons we are chiefly concerned with in this article, one-loop quantum effects modify the kinetic energy terms so that they are not canonical; this shifts the characteristic surfaces, and so shifts the signal speed so that it is no longer equal to c .

¹⁴Note that, if a causal link between two events could only be established on the basis of their temporal order, the notions of cause and effect would be problematic in theories (such as, *e.g.*, Newtonian gravity) which admit instantaneous action at a distance.

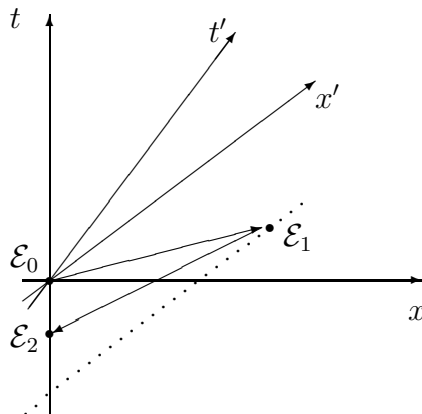


Figure 2: A causal paradox using tachyons. The dotted line represents the set of events which are simultaneous with \mathcal{E}_1 according to the reference frame \mathcal{K}' . The tachyonic signal from \mathcal{E}_1 to \mathcal{E}_2 travels to the future with respect to \mathcal{K}' , and to the past with respect to \mathcal{K} .

should not be more disturbing than the jet lag experienced by travellers because of the peculiar way clocks are set on the Earth.¹⁵ Note, however, that if \mathcal{E}_1 causes \mathcal{E}_2 , there is always at least one frame in which \mathcal{E}_2 does not happen earlier than \mathcal{E}_1 .

The inversion of the time ordering for two events connected by the propagation of a tachyon is therefore not, by itself, a difficulty. However, unless some restriction is imposed on the type of propagation, it is potentially a source of paradoxes, as it can lead to situations where two events are *timelike* related, and yet the cause follows the effect. A typical argument is the following, sometimes picturesquely referred to as the “tachyonic anti-telephone” [44].

Suppose that, in an inertial frame \mathcal{K} , a tachyon is emitted at $t_0 = 0$, $x_0 = 0$ (event \mathcal{E}_0 in figure 2), and received at an event \mathcal{E}_1 with $t_1 > 0$. It is always possible to find another inertial frame \mathcal{K}' , in the configuration considered in section 2.1, such that $t'_0 = 0$, $x'_0 = 0$, and $t'_1 < 0$. Now, suppose that at the event \mathcal{E}_1 a second tachyon, that travels to the future with respect to \mathcal{K}' and to the past with respect to \mathcal{K} , is sent toward the origin. This reaches the spatial origin of \mathcal{K} (event \mathcal{E}_2) at a time $t_2 < 0$. We can arrange the experiments in such a way that \mathcal{E}_0 causes \mathcal{E}_1 which, in turn, causes \mathcal{E}_2 . Therefore, \mathcal{E}_0 causes \mathcal{E}_2 , which is a paradoxical result because, since these two events are timelike related and $t_0 > t_2$, \mathcal{E}_0 follows \mathcal{E}_2 in *all* reference frames. More elaborate versions of this paradox, based on the particular use of Scharnhorst photons, have been presented in [20].

It is obvious from the description above that paradoxes of this type require not only that tachyons exist, but also that, given an *arbitrary* reference frame, it is always possible to send a tachyon backward in time in that frame. Obviously, there can be no paradox if, in one *particular* reference frame, tachyons can only propagate forward in time.¹⁶ This

¹⁵The only case in which an inversion of the time ordering could really be problematic for causality is when the events happen at the same place in one reference frame. This possibility is excluded for transformations that fulfill the pre-causality condition of section 2.1, including of course the Lorentz ones.

¹⁶Of course such a restriction is anathema in the standard approach to special relativity since it picks

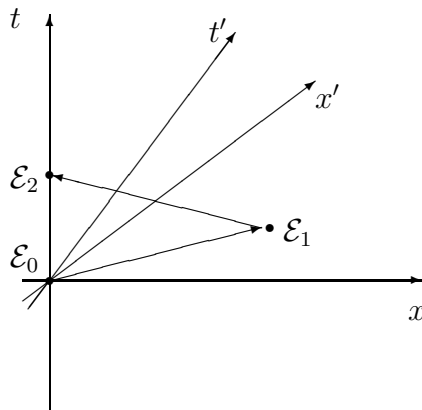


Figure 3: Tachyon propagation without causal paradoxes. Both signals travel to the future in at least one reference frame.

situation is exemplified in figure 3, which shows a chain of events similar to the one of figure 2, with the only difference that now the tachyon has a fixed speed with respect to the reference frame \mathcal{K} . This time, the event \mathcal{E}_2 takes place *after* the event \mathcal{E}_0 in \mathcal{K} . Moreover, since the two events are timelike related, \mathcal{E}_2 follows \mathcal{E}_0 in *all* frames. Thus, there is no paradox, although tachyons are used to send signals. In the next subsection, we show that the anomalous photons that give rise to the Scharnhorst effect are precisely “benign” tachyons of this type. (See also [45] for arguments along similar lines.) More radical proposals to solve the paradoxes presented by *any sort* of faster-than- c particles can be found in [11, 43] and references therein. Our goal is much less ambitious, as we restrict our consideration to Scharnhorst photons only.

3.2 Scharnhorst photons preserve causality

In order to show that Scharnhorst photons do not lead to causal paradoxes, we first introduce an effective metric describing photon propagation in the Casimir vacuum. We then show that even though the light-cones are wider than those of the Minkowski metric, the spacetime is nevertheless stably causal, which prevents causal paradoxes from occurring. If there are multiple pairs of plates in relative motion, the situation is trickier, and we analyze it in terms of Hawking’s chronology protection conjecture.

3.2.1 Effective metric

The propagation of light signals between perfectly conducting plates is described by the dispersion relation $\gamma^{\mu\nu} k_\mu k_\nu = 0$, where k_μ is the wave vector (actually, a one-form), and the coefficients $\gamma^{\mu\nu}$ have the form [5]

$$\gamma^{\mu\nu} = \eta^{\mu\nu} + \xi n^\mu n^\nu, \quad (3.2)$$

out a preferred frame. However if you have good physical reasons for picking out a preferred frame (*e.g.*, the rest frame of the Casimir plates) this sort of restriction can make good physical sense.

where $\eta^{\mu\nu}$ is the inverse of the Minkowski metric, n^μ is the unit spacelike vector orthogonal to the plates, and ξ is a function. We consider the parallel plates to be orthogonal to the x axis, so $n^\mu = (0, 1, 0, 0)$.¹⁷ The size of the corrections to propagation in the absence of boundaries can be obtained computing ξ to order α^2 , where α is the fine structure constant. Using the Euler–Heisenberg Lagrangian one gets [5]

$$\xi = \frac{11 \pi^2 \alpha^2}{4050 a^4 m_e^4} \approx 4.36 \times 10^{-32} \left(\frac{10^{-6} \text{ m}}{a} \right)^4, \quad (3.3)$$

where a is the distance between the plates in their reference frame and m_e is the electron mass. Of course, so tiny a figure means that detecting the effect is a task totally beyond present-day technology. However, as we shall now show, the fact that ξ is positive leads to a speed of propagation (slightly) larger than c , which constitute a potential threat for causality, no matter how small the deviation is.

The $\gamma^{\mu\nu}$ can be interpreted as coefficients of an effective inverse metric [5, 16]. The metric itself is thus obtained by inverting $\gamma^{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{\xi}{1 + \xi} n_\mu n_\nu. \quad (3.4)$$

(Warning: We keep raising and lowering indices using the Minkowski metric $\eta_{\mu\nu}$ and its inverse $\eta^{\mu\nu}$. In particular, note that $g_{\mu\nu} \neq \eta_{\mu\rho} \eta_{\nu\sigma} \gamma^{\rho\sigma}$.) Since ξ is positive, the light cone associated to the effective metric $g_{\mu\nu}$ is slightly wider, in the direction orthogonal to the plates, than the one corresponding with $\eta_{\mu\nu}$. This implies that light travels at a speed c_{light} slightly larger than c , except in the case of propagation parallel to the plates.

The exact value of this speed in the rest frame of the plates can be obtained by considering the way a light signal is actually sent from a point P in space to another point Q . A pulse at P originates a wavefront that propagates outwards and eventually reaches Q . The speed of light in the direction PQ is then given by the ratio

$$\frac{\text{distance between } P \text{ and } Q}{\text{time taken by the wavefront to reach } Q}.$$

(Of course, the speed of light is independent of position, because $g_{\mu\nu}$ turns out to have constant coefficients.) Assuming that the pulse starts at $t = 0$ at the point P with coordinates $(0, 0, 0)$, the wavefront is described by the equation

$$S(t, \mathbf{x}) = -ct + (g_{ij} x^i x^j)^{1/2} = 0, \quad (3.5)$$

where Latin indices run from 1 to 3, and the function S — the eikonal — is a solution of

$$\gamma^{\mu\nu} \partial_\mu S \partial_\nu S = 0. \quad (3.6)$$

Thus, the time required for propagation from P to the point Q with coordinates (x, y, z) is

$$ct = (g_{ij} x^i x^j)^{1/2} = \mathbf{x}^2 - \frac{\xi}{1 + \xi} (\mathbf{n} \cdot \mathbf{x})^2 = |\mathbf{x}| \left(1 - \frac{\xi}{1 + \xi} \cos^2 \varphi \right)^{1/2}, \quad (3.7)$$

¹⁷ Note the difference with respect to reference [5], where the plates were taken orthogonal to the z axes.

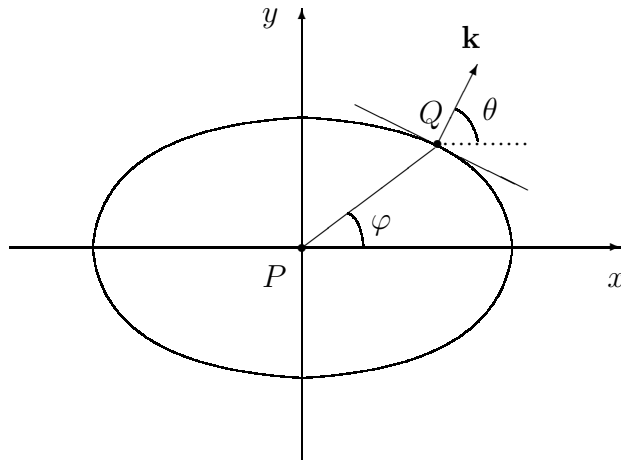


Figure 4: Wavefront (or line of constant phase) for a light signal emitted at the point P . Note that the wave vector \mathbf{k} forms a non-vanishing angle $\theta - \varphi$ with the direction of propagation.

where φ is the angle that PQ forms with the normal to the plates, \mathbf{n} . The speed of propagation for light in the direction PQ is then

$$c_{\text{light}}(\varphi) = \frac{|\mathbf{x}|}{t} = c \left(\frac{1 + \xi}{1 + \xi \sin^2 \varphi} \right)^{1/2}. \quad (3.8)$$

Note that $c_{\text{light}}(\varphi)$ is different from the phase velocity [5]

$$v_{\text{phase}}(\varphi) = c \left(1 + \xi \cos^2 \varphi \right)^{1/2} \quad (3.9)$$

corresponding to an angle φ between the wave vector \mathbf{k} and \mathbf{n} . This discrepancy is due to the fact that the surfaces of constant phase are ellipsoidal rather than spherical, which leads to a tilt between the direction of propagation PQ and the wave vector $\mathbf{k} = \nabla S$ (see figure 4).

The Scharnhorst photons travel at a definite speed in any given reference frame. This is a trivial consequence of the previous result, that they have a well-defined speed $c_{\text{light}}(\varphi)$ in the reference frame at rest with respect to the plates, and of the relativistic law for the composition of velocities. However, one can check this explicitly using the description of photon propagation given by the effective metric (3.4).

For two events with coordinates $\{x^\mu\}$ and $\{x^\mu + \delta x^\mu\}$, lying along the worldline of a photon, we have $g_{\mu\nu} \delta x^\mu \delta x^\nu = 0$. An observer with four-velocity u^μ will decompose the spacetime displacement δx^μ into a time lapse

$$\delta\tau = -\frac{1}{c} \eta_{\mu\nu} u^\mu \delta x^\nu \quad (3.10)$$

and a space displacement

$$\delta\chi = \left(\eta_{\mu\nu} \delta x^\mu \delta x^\nu + c^2 \delta\tau^2 \right)^{1/2}, \quad (3.11)$$

respectively. The photon speed $c_{\text{light}}^{(u)}$ with respect to the observer u^μ is given by

$$c_{\text{light}}^{(u)} = \lim_{\delta\tau \rightarrow 0} \frac{\delta\chi}{\delta\tau} = c \lim_{\delta\tau \rightarrow 0} \left[1 + \frac{\xi}{1 + \xi} \left(\frac{n_\mu \delta x^\mu}{u_\nu \delta x^\nu} \right)^2 \right]^{1/2}, \quad (3.12)$$

where the relationship

$$\eta_{\mu\nu} \delta x^\mu \delta x^\nu = \frac{\xi}{1 + \xi} (n_\mu \delta x^\mu)^2, \quad (3.13)$$

which follows from (3.4), has been used. The important points about equation (3.12) are that $c_{\text{light}}^{(u)} \geq c$ always, and that $c_{\text{light}}^{(u)}$ has only one value for each observer u^μ . The latter feature prevents the possibility of causal paradoxes, which require signals that travel with the *same* speed greater than c in two different reference frames.

3.2.2 Stable causality

Adopting the formalism of the effective metric (3.4) allows us to borrow many of the notions of general relativity, and to conclude in a more straightforward way that Scharnhorst photons cannot violate causality. In particular, since in general relativity the global causal structure is not specified by the field equations, considerable thought has gone into how to characterize causality [46]. We only need one key definition, and one key textbook result.

Definition: Stable causality. A spacetime is said to be stably causal if and only if it possesses a Lorentzian metric $g_{\mu\nu}$ and a globally defined scalar function t such that $\nabla_\mu t$ is everywhere non-zero and timelike with respect to $g_{\mu\nu}$. (See reference [46], p. 198.)

Theorem: A stably causal spacetime possesses no closed timelike curves, and no closed null curves. (See reference [46], p. 199.)

The relevance of this definition and theorem is that spacetime, endowed with the Minkowski metric (3.1) outside the plates, and with the effective metric (3.4) inside the plates, is stably causal. Indeed, using the time coordinate in the rest frame of the plates as the globally defined t , then $\nabla_\mu t$ is everywhere non-zero and timelike. Therefore causality is safe.

This is enough to guarantee that you can never encounter a causality violation if you only have a single pair of Casimir plates to deal with, or even multiple plate pairs that do not intersect and do not move with respect to each other. Multiple pairs of Casimir plates in relative motion will be a little trickier. Far in the past, when the pairs are well separated, there is certainly no causality violation and the question arises as to what happens as the various pairs of plates approach each other.

3.2.3 A paradox with multiple pairs of moving plates?

Suppose we have *two* pairs of Casimir plates, moving with relative speed v_{relative} : Let the usual speed of light be c and the speed of the Scharnhorst photons be $c_{\text{light}} > c$. Then the

usual textbook exercises presented in elementary courses of special relativity would seem to indicate the *risk* of causality violations whenever ¹⁸

$$v_{\text{relative}} c_{\text{light}} > c^2. \quad (3.14)$$

But before we deduce the existence of an actual logical paradox we should look more carefully at the spacetime diagram.

Suppose the plates are truly infinite in extent, as they should be for the Casimir vacuum to be exact. Then to generate the causal paradox of figure 2 the entire closed timelike loop $\mathcal{E}_0 \rightarrow \mathcal{E}_1 \rightarrow \mathcal{E}_2 \rightarrow \mathcal{E}_0$ must lie inside *both* pairs of Casimir plates. That is, the two Scharnhorst vacua must be inter-penetrating, in particular the conducting plates required to set up the Casimir vacuum must be able to pass through each other without affecting each other. But in that case, the symmetries leading to the effective metric in equation (3.4) have been violated, so there is no reason to believe that the effective metric will look anything like (3.4), and there is no reason to believe that any closed timelike loop forms.¹⁹

We could try to be a little more realistic by taking two pairs of half-infinite plates. To be specific let one pair of half-infinite plates be confined to the region $x > 0$, while the second pair is confined to the region $x < 0$. Then one could try to set up a causal paradox by, for instance, sending messages to the left using the vacuum corresponding to the $x > 0$ plates, and sending messages to the right using the $x < 0$ plates. (To completely close the loop we would also need to send two messages along the x axis, to get from the $x > 0$ vacuum to the $x < 0$ vacuum and back.)

The problem in this case is edge effects: the effective metric given in (3.4) is only expected to be a good approximation far away from the edge of the plates, while well outside the plates the effective metric should approach that of Minkowski space. Near the edge of the plates the effective metric is impossible to calculate, and the situation only gets worse when two pairs of half-infinite plates pass each other with a grazing not-quite collision. It is certainly clear that the simple naive result of equation (3.14) should not be trusted.

The present arguments do not guarantee the total absence of causality violations, but they do demonstrate that the most naive estimates of the causality violating regime are likely to be grossly misleading.

3.2.4 Chronology protection

To actually demonstrate that causality violation is excluded, at least if we stay within the realm of semiclassical quantum field theory (and this is the underlying approximation made to even set up the entire formalism), we invoke the notion of “chronology protection” in the sense of Hawking [47, 48, 49, 50].

¹⁸This condition on the velocities simply comes from the Lorentz transformation of the time coordinate

$$t \rightarrow t' = \gamma (t - v_{\text{relative}} x / c^2).$$

Setting $x = c_{\text{light}} t$, and asking for $t' < 0$, implies $v_{\text{relative}} c_{\text{light}} > c^2$. One then applies the same logic to the return trip to find a necessary condition for the existence of closed timelike curves.

¹⁹In particular the region containing the putative closed timelike loop $\mathcal{E}_0 \rightarrow \mathcal{E}_1 \rightarrow \mathcal{E}_2 \rightarrow \mathcal{E}_0$ is simultaneously subject to two different and incompatible effective metrics, so the logic that at first glance seems to lead to closed timelike loops is actually internally inconsistent.

The point is that when the two pairs of plates are well separated, long before the two pairs approach each other, they are individually stably causal and there is no risk of closed timelike curves — the “initial conditions” are that the universe starts out with sensible causality conditions. So if a region of closed timelike curves forms as the two pairs of plates approach each other, that region must have a boundary, and there must be a “first” closed null curve.

Hawking has argued that the appearance of the first closed null curve will lead to uncontrollable singularities in the renormalized quantum stress-energy as vacuum fluctuations pile up on top of each other [47]. (Roughly speaking, because of the existence of a closed null curve, a single photon [real or virtual] could meet up with itself and coherently reinforce itself an infinite number of times.) Though Hawking’s argument was presented in the context of the causal problems typically expected to arise in Lorentzian wormholes, the argument is in fact generic to any type of “chronology horizon”.

A second, slightly different, version of chronology protection has been formulated by Kay, Radzikowski, and Wald [48]. Technically, they demonstrated that at the chronology horizon the two-point function (Green function) fails to be of Hadamard form. As a consequence they could argue that the renormalized stress tensor does not even make sense at the chronology horizon, and that the entire formalism of quantum field theory on a fixed background spacetime suffers total failure. (Note that in this approach the stress-energy tensor may remain bounded in the vicinity of the chronology horizon, though it is guaranteed to be ill-defined at the chronology horizon itself. See also [49].)

One of the present authors has further argued that in the vicinity of any chronology horizon there is an invariantly defined “reliability horizon” beyond which Planck scale physics comes into play [50] — the region characterized by the existence of closed space-like geodesics shorter than one Planck length is a region subject to violent Planck-scale fluctuations, even if the expectation value of the stress-energy is small.

All these variants on the idea of chronology protection agree on one thing: If the geometry is to remain well-defined so that semiclassical physics makes sense, which is certainly necessary to even define the basic precursors leading to the notion of causality, then the chronology horizon must fail to form. Indeed, if the chronology horizon somehow does manage to form, then causality paradoxes are the least of your worries since you are automatically driven into a regime where Planck-scale quantum gravity holds sway.

4 Conclusion

In this paper, we have presented a critical assessment regarding the possibility, from the point of view of the basic physical principles of relativity and causality, of faster-than- c signalling. Our analysis is motivated by some recent theoretical predictions of “superluminal” photon propagation [2, 3, 14]. We have shown that such effects are kinematically compatible with special relativity, because the latter requires only the existence of an invariant speed, not necessarily a maximum one. Also, they do not lead to causal paradoxes, which can arise only from tachyons whose speed has no fixed value in a given reference frame. Perhaps surprisingly, it is the soft breaking of Lorentz invariance, which always occurs in these effects, that fixes the value of the speed of the faster-than- c photons in a specific frame and thus saves causality. Such a breaking is, however, due to the choice of a vacuum state and does not extend to the invariance group of the fundamental physical

laws, which still remains the usual $C = c$ Lorentz group.

Appendix A: Alternative kinematical groups?

The derivation of Lorentz transformations *à la* von Ignatowsky, on which our analysis in section 2.1 relies, shows how robust the structure of the Lorentz group is. In order to obtain different transformations, at least one of the hypotheses (i)–(iii) listed at the beginning of section 2.1 must fail to apply. This remark is relevant in the context of the now fashionable attempts at replacing the Lorentz group by a different set of transformations, in order to provide a kinematical basis for a new physics that is not Lorentz-invariant at high energies [51], which might be needed to account for the observed overabundance of cosmic-ray TeV photons and the missing GZK cut-off in a natural way. (See *e.g.*, [52] and relevant references therein.) The hypothesis that is most likely to fail is (ii) — the relativity principle. However, since (ii) enters von Ignatowsky’s argument merely as the requirement that the set of transformations between inertial frames form a group, it is obvious — indeed, tautological — that no group structure can emerge if one rejects it. In fact, it is a result of the analysis performed in references [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] that the Lorentz and Galilei transformations are the *only* ones that satisfy (i)–(iii).

We must add a caveat, though. In addition to (i)–(iii) there are other, more fundamental, hypotheses underlying *any* derivation of the Lorentz transformations. These are the (usually implicit) assumptions that measurements of time intervals are expressed by arbitrarily small real numbers, that it is possible to describe space in terms of Cartesian frames, and that frames can move with respect to each other at an arbitrary speed $v < C$. The first two assumptions require that there is no fundamental scale in Nature. The third one is a consequence of combining scale invariance with the relativity principle. However, if there were a fundamental length scale in the physical universe, say l_0 , under which no measurement could be performed, it would be meaningless to consider Lorentz transformations at speeds arbitrarily close to C . Indeed, consider two frames \mathcal{K} and \mathcal{K}' which define a length unit following the same procedure — a hypothesis implicit in special relativistic kinematics, as we discussed in section 2.3. It is always possible to find a relative speed which is so large that the unit of \mathcal{K}' as measured from \mathcal{K} turns out to be smaller than l_0 , in which case no comparison between measurements can be performed. Thus, it might be that, although hypotheses (i)–(iii) are still satisfied, von Ignatowsky’s proof of the necessity of the Lorentz group is nevertheless vitiated by rejecting some assumption which is even more basic. This leaves open the possibility that non-Lorentzian kinematical groups could somehow be developed.

Appendix B: Maximum speed and negative energy densities

Although not crucial for the consistency of special relativity, it would nevertheless be important to understand whether there are physical bounds for the speed of signals. The possibility of superluminal effects is usually associated with special conditions on the expectation value of the stress-energy-momentum tensor for the quantum fields in the modified vacuum state. Indeed, in general homogeneous non-trivial vacua, the inverse

effective metric for photon propagation takes (at lowest nontrivial order) the form

$$\gamma^{\mu\nu} = \eta^{\mu\nu} - Q\langle T^{\mu\nu} \rangle,$$

where Q is, in general, a function of the variables and parameters of the Lagrangian [16]. In the particular case of the Scharnhorst effect Q turns out to be (to order α^2) a positive constant. It is then clear from the dispersion relation $\gamma^{\mu\nu}k_\mu k_\nu = 0$, together with the above, that in order for the modified light cones to be wider than those of $\eta_{\mu\nu}$, one must have $\langle T^{\mu\nu} \rangle k_\mu k_\nu < 0$. Since k^μ is a null vector with respect to the Minkowski metric, it follows that the Scharnhorst effect implies a violation of the null energy condition.

If this circumstance were generic, *i.e.*, if the increase of the speed of light were always associated with the presence of negative energy densities, due to vacuum polarization [15, 16], a possible maximum velocity should correspond to a maximum negative energy density. The problem becomes then whether there is a lower bound for the energy density. One can envisage several possibilities that would lead to this conclusion: quantum field theory itself could break down when the vacuum energy density is too high (in absolute value), or something like the Ford–Roman quantum inequalities [53] could apply.²⁰ It is also conceivable that non standard geometrical terms may appear in higher order corrections to the Casimir effect and lead, at ultra-short separations of the plates, to positive energy densities. Of course, in this latter case the maximum speed would be a geometry- (as well as vacuum-) dependent quantity, rather than a fixed constant of nature. Any of these possibilities, or any other one giving a lower bound on the energy density, would in turn lead to an upper bound for the photon speed.

Appendix C: Reprise — What is C ?

Between Casimir plates, the internal constitution of standards of time and distance based on the electromagnetic interaction is certainly influenced by the same processes that lead to the Scharnhorst effect. This might turn into an “invisibility” of the effect itself, similarly to what happens, according to Lorentz, in a reference frame that moves with respect to the æther, when dynamical contraction of rods and slowing of clocks do not allow one to measure the “true” speed of light [40]. In other words, measurements performed using an equipment based on electromagnetism might lead to the empirical result that light travels always at the speed $c = 2.99792458 \times 10^8 \text{ m s}^{-1}$, even though our theories imply that the speed is c_{light} , not necessarily identical to c . However, one can use units which are unaffected by the presence of plates — for example, based on gravitational physics — which would still measure c as the invariant speed, thus making possible to detect the Scharnhorst effect, at least in principle.

Nevertheless, the possibility that it is c_{light} , rather than c , the velocity that would turn out to be invariant by measurements based on electromagnetic standards, seems to suggest that, as far as such measurements are concerned, one should consider a Lorentz group with $C = c_{\text{light}}$. However, since $c_{\text{light}} \neq c_{\text{gravity}}$, this Lorentz group would not be the one appropriate to define local Lorentz invariance in general relativity. (Of course, the same situation would occur if photons and gravitons were travelling at different speeds.) Furthermore, there would be two possible candidates to the role of invariant speed C in

²⁰ If the amount of energy which can be confined in a given spacetime volume $V^{(4)}$ is limited by the uncertainty principle, one would dimensionally expect a relationship of the type $\langle \rho \rangle > -\hbar/V^{(4)}$.

special relativity. From the arguments in section 2.3 it follows that one cannot choose *a priori* whether $C = c_{\text{light}}$ or $C = c_{\text{gravity}}$, because the actual value of C is related to the behavior of real clocks and rods. Likely, electromagnetic clocks/rods will correspond to $C = c_{\text{light}}$, while gravitational clocks/rods should lead to $C = c_{\text{gravity}}$. Since it seems hard to live with *two* relativity theories, the most natural step to take in this case would be perhaps to reconsider the role of Lorentz invariance, regarding it as a symmetry property of specific theories rather than as a fundamental meta-principle of all physics.

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