Aircraft Impact Damage

Tomasz Wierzbicki
Professor of Applied Mechanics, MIT

Liang Xue
Ph.D. Candidate of Ocean Engineering, MIT

Meg Hendry-Brogan
Undergraduate student of Ocean Engineering, MIT

Abstract

The "post-September 11th" structural engineer, while feeling the remorse and confusion that every other American has dealt with, is also privileged with the immense education an analysis of the WTC collapse can provide. A newly found understanding for impact dynamics and failure of very large systems, as well as a comprehensive grasp of the brevity accompanying safety considerations in construction projects, will be present in industrial practice from now on. The research into the World Trade Center Towers collapse following the initial fact-gathering phase is now beginning the more ambitious tasks of reconstructing various stages of the damage and destruction of the Twin Towers. Currently, or at least as current as this paper, the FEMA/ASCE team has just released their report, [1], and an independent investigation is being conducted by the National Science Foundation study group. Preparations are also underway to launch a new program aimed at a producing a detailed simulation of the aircraft impact damage, fire damage, and the total collapse of the buildings. This work is led and coordinated by the National Institute of Standards and Technology.

This article was completed prior to the public release of the FEMA/ASCE report, therefore only the generally accessible information from the media and literature were used in the analysis. The facts documenting the first phase of the main objective of the present research is to predict the amount of internal structural damage that occurred within the Towers upon the aircraft impact and that was hardly visible from the outside. Attention is focused on three main structural components of the Towers, i.e., a lattice of exterior columns, complex floor truss assemblies, and the core load-bearing structure. A thorough understanding of failure mechanisms and the extent of damage done when a high speed aircraft impacts a large-scale structural system is a prerequisite for undertaking the next stage of the analysis, which is the weakening effect of fire and finally the self-distracting implosion of both Towers. The airplane itself, built as an assemblage of ring and stringer-stiffened panels, was also subjected

to gradual break-up and dis integration. The problem of interactive failure and fragmentation of two deformable and fracturing bodies, i.e., the aluminum airframe and steel structure, has not been addressed in the literature. Therefore, the question remains whether an estimate can be made on the internal damage of the building before the necessary computational tools are developed and small and full-scale tests are conducted? The answer to this question is yes, only if proper use is made of a few basic laws of mechanics. The method that is chosen here involves a logical progression from first principles to a recreation of the complex series of failure models, which set the stage for each Tower's final collapse. There are three basic principles of mechanics that are invoked in the present analysis

- conservation of energy
- conservation of linear momentum
- principle of virtual work

Each of the above laws of mechanics applies to a different scale. The energy conservation applies to the global scale of the entire aircraft and the affected parts of the building. It is expressed through the following equation

$$E_{kinetic} = E_{plane} + E_{external\_column} + E_{floor} + E_{core} \qquad (1)$$

This equation says that the initial kinetic energy of the aircraft $E_{kinetic}$ (which is known) is converted into the energy dissipated by plastic deformation and fracture of four constituents of the collision problem, i.e., the airframe itself E_{plane}, the external column $E_{external\_column}$, the floors E_{floor}, and the core structure E_{core}. Some energy is also lost by friction and is converted into the elastic vibration of the entire building. These two contributions are small and will be neglected in the present simplified analysis.

Taking the estimated airplane mass at the point of impact to be $M = 127$ tons and the impact velocity of $V_o = 240\text{m/s}$, the energy of the striking aircraft was $E_{kinetic} = 3658\text{MJ}$. In the main body of this article, estimates are made on each comp onent of the dissipated energy on the right hand side of Eq.(1). For each structural element, plastic energy is dissipated thought two mechanisms. The first mechanism is plastic deformation through the tensile tearing or shear plugging mode. This portion of the energy can be clearly distinguished by looking at the color-coded strain fields in computer simulation and therefore we call it "*visible*" energy. The other component of the energy loss is associated with the momentum transfer, which is difficult to see on the output of computer simulation. Accordingly, we call that contribution as the "*invisible*" energy. Depending on the impact velocity, relative magnitude of both energies could be different, but they should both be considered in a rigorous analysis of an inelastic impact.

The external columns were impacted at a very high speed and the process is controlled mainly by local inertia. As the fuselage and wings cut through the steel facade of the Towers, the affected portions of the column sheared off. It was found that the momentum transfer between the airframe and the first barrier of external columns was responsible for most of the energy dissipated in this phase. The energy to shear off the column constituted only a small fraction of that energy. A more exact calculation performed in Ref. [2] give a slightly larger value $E_{external\_column} = 26\text{MJ}$.

The floors and floor trusses were the next barrier to overcome. The floor trusses consisted of hundreds of beam-like tubular members. At this stage of the analysis it was impossible to develop a detailed computational model of this complex assembly. Therefore the entire volume of steel used by the floors was lumped into a uniform steel plate of the equivalent thickness. It was estimated that loss of kinetic energy to plow the airframe through the model structure was $E_{floor} = 1221\text{MJ}$ for North Tower and $E_{floor} = 1040\text{MJ}$ for the South

Tower. As for the airplane itself, the process of disintegration of the fuselage and wings started immediately during the entry into the wall of the exterior columns and it continued as the floors were cut and ripped apart.

Research available on high speed aircraft impacts into rigid and/or deformable bodies is limited in scope and pertains largely to reinforced concrete walls that protect nuclear power stations. The process of interaction of the airframe with a tube-like or cage-type steel structure is different. In the present calculation simpler models to crush and slice the fuselage and damage the wings into the central spar, open beam sections, ribs, and skins are used.

It was hoped that pieces of the aircraft were retrieved from "Ground Zero" to find the average size of the fragments. This will help to determine the actual energy expended through the breakup of the fuselage. The FEMA/ASCE failed to provide this information. Another source of inaccuracy in the determination of energy dissipated in failing the aircraft is the uncertainty presented by the impact orientation. The diameter of the plane is, in fact, larger than the length between floors, but different interactions will take place based on the orientation of the aircraft floors and wings with respect to the major axis of the external columns of each Tower. The calculation used to determine E_{plane} in this analysis takes these two uncertainties into consideration and attempts to make up for this error contribution by carefully superposing the energy dissipated through each step of the plane fragmentation and fracture. The calculations are completed taking both deformable and rigid body mechanics into account. Obvious rigid components, like the engines, weren't considered deformable in any part of the calculation. In the end, the lower bound on the energy expanded to distressing the aircraft was found to be $E_{plane} = 962\text{MJ}$.

The energy to be dissipated by the core structure is the difference between the total energy introduced into the Towers $E_{kinetic}$ and the energies lost on damaging the exterior columns, floors, and the aircraft itself. From Eq.(1) this energy was found to be $E_{core} = 1630\text{MJ}$ for the South Tower and $E_{core} = 141\text{MJ}$ for the North Tower. There are a lot of uncertainties as to what happened to the core structure under such high energy input. One could envisage partial damage (bending) of many columns or complete damage (severance) of fewer columns. By the time the pile of debris from the airplane and floors the load on core column would probably be much more distributed favoring severe bending rather than of core columns cutting. It is estimated that 7 to 20 core columns were destroyed or severely bent in the South Tower while only 4 to 12 core columns were ruptured in the North Tower. These initial estimates can be easily adjusted once more precise information on the geometry, material, and impact condition become available.

At the end of this article several important factors pertinent to the global collapse of buildings are discussed. However, a more precise sequence of events which trigger the ultimate implosion of buildings is left to a future continuation of this research.

The first draft of this article was actually completed in February and printed as Report #74 of the Impact and Crashworthiness Lab. Subsequently, four new reports on analytical and numerical analyses of the aircraft impact problem have been completed [10-14]. The results of these reports, whenever necessary, have been incorporated into the updated version of Report #74 which constitutes the present article.

1. Introduction

On January 28, 1986 the space shuttle Challenger exploded in mid air and plunged into the ocean at a terminal speed of 80 m/s (180 mph), shattering the crew compartment and killing everyone in it. NASA and the Presidential Commission carried out an investigation that revealed the root cause of the accident. However, the report failed to provide a reconstruction of the three stages of the accident (i.e. mid air explosion, free fall and water impact). One of the present authors (TW) carried out a separate investigation of the space shuttle disaster and presented a detailed analysis of each of the above stages of the accident in the open literature [3-5].

On September 11, 2001 another disaster of far greater proportion struck the nation. Officials immediately began clearing the site of the accident, and collecting data. As of today, six months after the accident, no step-by-step reconstruction of all the factors leading to the collapse of the WTC Towers has been released. However, there has been an ongoing debate in the academic community over many of the key elements integral to a firm structural failure theory [6]. The present analysis uses the limited, publicly available data from the crash site, to reinforce certain first principles of mechanics in order to abstract upon the events of September 11th. The recently release FEMA/ASCE report add very little into the understanding of the aircraft impact damage and focus mainly on the global collapse of the Twin Towers and the adjacent buildings. Should new information, coming from such sources as a Nation Science Foundation study group, provide additional relevant data, our analysis should be quickly modified with little additional effort because of the character of our close-form solution. Therefore, we believe that the underlying methodology employed below transcends a mere reconstruction of the crash, but more importantly provides a much-needed understanding of the structural failure processes that characterize high velocity aircraft or missile impacts with large civilian or military installations.

2. Objects and approach

The functional objective of this article is to make educated predictions of the internal structural damage that occurred within the towers and that was hardly, if at all, visible to the observer. These "invisible" parts of the buildings, i.e. the complex floor truss assemblies and the core load-bearing structure, shown in Figure 1, comprise an integral part of any analysis into the ultimate collapse of the towers. They are the elements of the collapse reconstruction that are lightly understood and highly speculated upon. This analysis attempts to achieve a higher understanding of this area of the collapse via complex, first-order modeling of the major components of the impact: the building and the plane.

From the television video clips of the accident, a terrifying truth comes to life. The airplanes collided with the buildings at a cruising speed, cut through the outer shell and disappeared inside the towers. No appreciable pieces of the airplanes were seen to fully penetrate the Towers and emerge on the other side. (In fact, according to the FEMA/ASCE report, part of the engine and landing gear as well as a small portion of fuselage penetrated the outside structure and fell a few blocks away.)
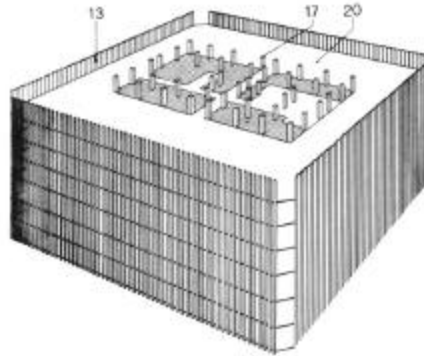
Figure 1. Double hollow tubes structures of the World Trade Center showing
exterior columns (13), floors (20) and core columns (17)

In the language of mechanics the above observation can be expressed via the statement of energy balance given by Eq.(1) where all the components entering Eq.(1) are listed below.

$E_{kinetic}$ is the kinetic energy of the airplane;

E_{plane} is the energy dissipated by the crushing and breakup of the aircraft;

$E_{external\_column}$ is the energy required to cut through the exterior columns;

E_{floor} is the energy dissipated by the floors;

E_{core} is the energy absorbed by the core column destruction.

In subsequent sections we will estimate all five different terms entering Eq.(1).

This is not an easy task because the relative contribution of various terms will depend on the activated failure modes and contact forces developed between different components of the airplane and the Towers. Both the airplane and the WTC Towers are built as closed or open, thin-walled, three-dimensional structures, which deform plastically, crush and crumble, fracture and break up into small pieces. Thus, whatever evidence remained has been burned in the 10-story high pile of debris.

What tools did the present team have at its disposal for accomplishing the stated objectives? To answer this question, one must place the local aircraft impact damage in the context of existing knowledge. A distinguishing feature of the attack on the Twin Towers was the high impact velocity that the airplanes had relative to the ground vehicle collisions extensively studied in the literature. A review of recent methods and results in the area of crashworthiness engineering can be found, for example, in references [7-9]. This class of problems is dominated by membrane and bending deformation of thin, shell-like structures accompanied by large displacement, rotation, and strain of material elements, as well as internal contact. Global inertia of structural members is important, but the effect of local inertia is negligible. Fracture is seldom a problem in crashworthiness engineering.

On the other end of the spectrum are projectile impacts into solid objects and/or thin sheets causing penetration and perforation. Here, fracture and local inertia play a major role, but projectiles are treated as rigid bodies when impacting thin-walled targets. Projectile impact velocities may exceed, by an order of magnitude, those that were encountered in the WTC Towers impact. For a review of the mechanics and physics of projectile impact, the reader is referred to excellent articles by Corbett et al [10] and Goldsmith [11].

Finally, there is vast literature, scattered over journal articles and conference proceedings dealing with the effect of explosion on structures, including fragmentation [12]. Some of the methods and results that are most relevant to the problem at hand are, unfortunately, classified.

Perhaps the most powerful tools available for solving structural impact problems are commercial Finite Element codes such as ABAQUS, LS-DYNA, ADINA, PAM-CRASH, etc. These codes can also handle fracture initiation and, to a limited extent, fracture propagation when the impacting bodies are discretized by tiny solid or shell elements. In a parallel study which is being conducted in the Impact and Crashworthiness Lab [13] fracture propagation was successfully simulated at the component level (see Figure 25). However, to be computationally efficient, large-scale structures must be discretized not by solid elements but by shell elements, which are larger in size but much fewer in numbers. When fracture and fragmentation is involved, the above codes can produce correct results for tension dominated fracture but may give large errors for shear dominated fracture [12].

For the purpose of the present analysis, an analytical approach will be used in which the simple solutions of several crushing and tearing problems involving thin walled structures will be combined into a coherent failure theory. Several reports have already been completed with involvement of the present authors addressing various stages of the fracture and fragmentation of exterior columns and wing structure, [2,14,15]. Therefore, we believe that our analyses are solidly rooted in the first principle of mechanics and therefore it will give a first order approximation of this enormously complex impact phenomenon.

2.1 Aircraft orientation and speed

Before a structural analysis can be made, initial conditions for the impact problem must be determined. This includes: aircraft speed, aircraft trajectory, point of impact, roll angle and orientation with respect to the floors. Most of the above data can be calculated from video clips available from CBS, see Figure 2, CNN, and the Washington Post. The two airplanes crushed into the Twin Towers were Boeing 767-200ER. The main geometric dimensions of a Boeing 767-200ER are

$$\text{Length: } l_f = 48.51\text{m}$$

$$\text{Wing span: } l_w = 47.57\text{m}$$

$$\text{Fuselage diameter: } D = 5.03\text{m}$$

$$\text{Max. take-off mass: } M = 179{,}330\text{kg}$$

Given that the maximum take-off mass of the airplane is 179,330 kg, that the airplane was not full of passengers (only 65 of 216 maximum capacity), and that the airplane was in the air for 50 minutes before it crashed into the WTC, the mass of the airplane is estimated to be $M = 127$ tons.

The independent assessment of the initial closing speed of the impacting aircrafts into the South Towers has been performed by present authors. A table below summarizes various estimation published in open literature.

Figure 2. The aircraft approaching the South Tower

Table 1. Impact speed of American Airline Flight 11 and United Airline Flight 175

	North Tower	South Tower
FEMA/ASCE Report [1]	210 m/s	264 m/s
Kausel [16]	192 m/s	240 m/s
Wald and Sack [17]	-	222 m/s
Present authors [14]	-	220-240 m/s

For the present calculation, it is assumed that impact velocities were 240 m/s and 200 m/s for the South and North Tower respectively. Hence, the initial kinetic energy of the airplane hitting the South Tower is

$$E_{South} = \frac{1}{2} MV_0^2 = 3658\text{MJ} \qquad (2)$$

The average estimated impact velocity of the United Airlines plane hitting the North Tower was $V_0 = 200 m/\sec$. The corresponding kinetic energy was much lower

$$E_{North} = 2540\text{MJ} \qquad (3)$$

The above calculations do take into account the kinetic energy of the fuel, however fail to provide for the energy introduced via the explosions or fires that the fuel sustained. In the present paper, we will be using the kinetic energy given by Eqs.(2) and (3).

The relative position of the aircraft with respect to the North and South Towers is shown (to scale) in Figure 3 and Figure 4 respectively.
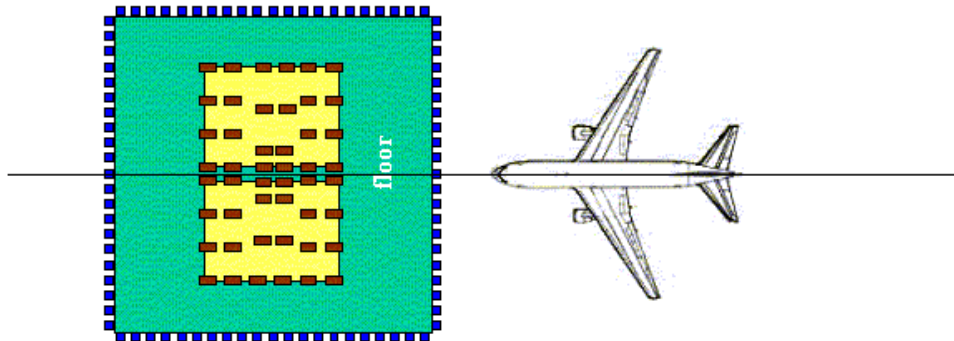

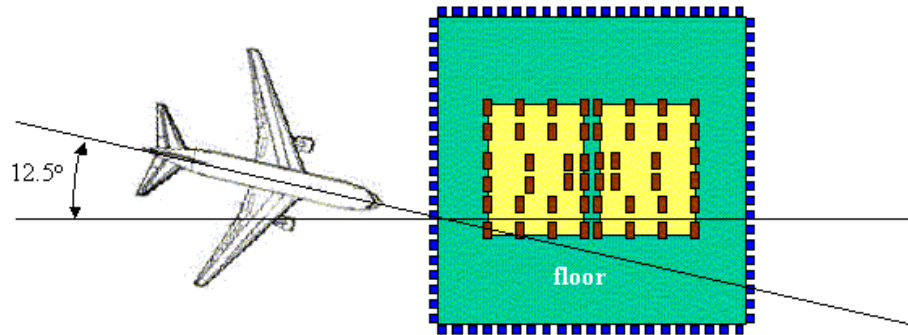Figure 3. Orientation of North Tower head-on impact

Figure 4. Orientation of South Tower oblique impact

Before colliding with the North and South Tower, the planes banked to the left and hit the Tower with a roll angle of approximately 26° and 35 ^{o}. This roll angle will have significant influence on the number of destroyed floors.
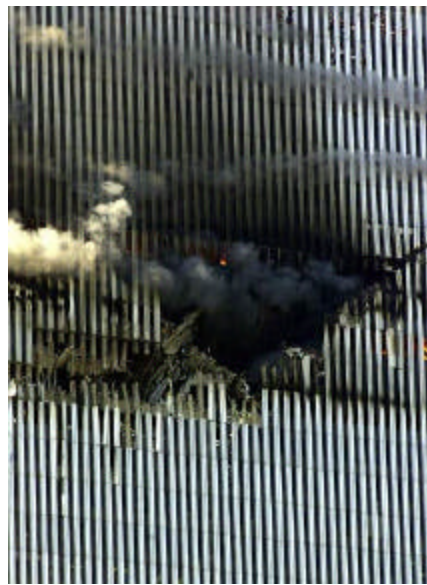

Figure 5. Damage to the exterior columns of the North Tower immediately after the impact.

The exact position of the longitudinal axis of symmetry of the plane with respect to a floor is unknown. However, we do know that the diameter of the fuselage (5.03m) was greater than the height between floors ($l = 3.7$m). Therefore, the fuselage will contact at least one floor, and more probably, two.

At the same time, the 3m diameter engines and the wings could easily fit between office floors. This will be most probably the case with the North Tower impact, which occurred with less roll angle.
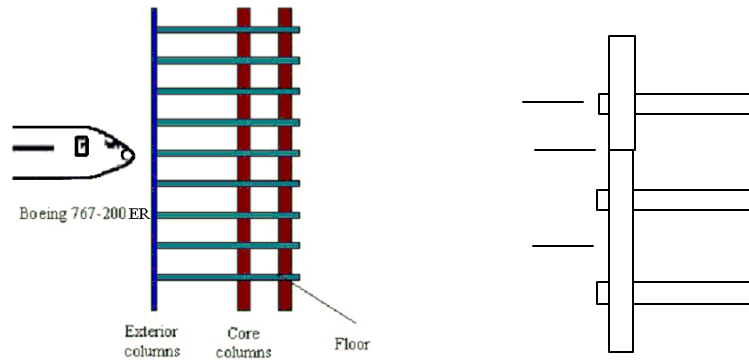
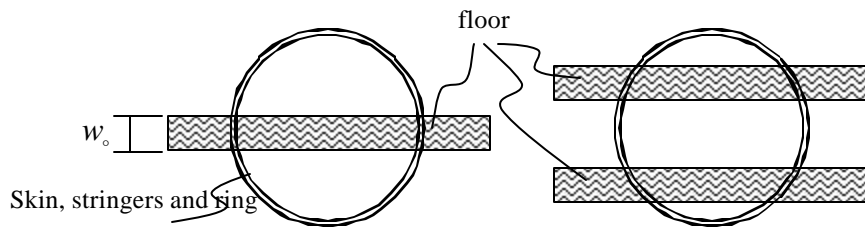Figure 6. Relative orientation of the aircraft and the floors

Figure 7. The 5-meter diameter of the fuselage can get engaged with one or two floors depending on the relative orientation

3. Aircraft failure

3.1 Modeling philosophy

In this engineering analysis, one must attempt to uncouple the problem of rigid vs. deformable body mechanics with respect to the airplane impact. The impact process is obviously a definite interaction between a very large stationary building and a small but fast moving airplane, both of which undergo considerable deformation. In order to make this problem mathematically tractable, some simplifying assumptions must be made. These assumptions essentially uncouple the impact interactions and then superpose them analytically. First, the building is treated as a rigid barrier and the airplane is considered deformable. Then the aircraft is treated as a rigid flying object, but the impacted structure is deformable.

The interaction between the impacting and impacted components is considered by monitoring the contact force and comparing the magnitudes of the forces required to instantaneously deform one or the other. The body that requires less force to collapse is treated as deformable, while the other is treated as rigid. This method was successfully used in the analysis of a collision between two ships [18]. The aircraft impact problem occurs at a much higher speed.

The first true "crash tests" of aircraft were conducted by Jerry Lederer at McCook Field, Ohio in 1924 [19]. Most pertinent to our research is the study initiated by Riera in 1968

for the Federal Aviation Administration [20] concerned with safety evaluation of the Three-Mile Island Nuclear Power Plant. Full-scale crash tests were conducted including the F-4D Phantom fighter [21] and DC-8 carrier [22]. Several research groups continued this line of research until recently [23-24].

One of the distinctive features of all the aircraft impact analyses performed for the nuclear industry is that all of the impacted structures (mostly dome shaped buildings) have been reinforced with 2m-thick concrete. Upon impact, there will be very little local damage to the dome in the form of crushing or scabbing and surface cracking of the concrete. Upon impact into high-rise buildings, the situation is different. The framework of beams, columns, and trusses could deform plastically and fracture. Because the contact area is small, these members, which are relatively narrow compared to the fuselage diameter, can cut and slice into main elements of the airframe before being broken themselves. Thus there is a complex iterative failure sequence between the two "opponents", building and airplane, that are of comparable strength.

3.2 Fuselage damage by steel framework

What happens to the airframe traveling with 240 m/s, encounters an absolutely rigid, but relatively narrow, obstacle such as steel columns or floors of the building? This analysis will require information on mass distribution and the structural details of a Boeing 767. Taking the data from the FEMA report, the mass of the airplane at the instant of impact is estimated to be equal 127 tons (including passenger aboard and 10,000 gallons of fuel). In the present level of approximation, the whole aircraft will be treated as being composed of three different types of structures: deformable fuselage, rigid engines and strong but crushable wings. The mass of the fuselage of a Boeing 767-300ER, which is 6.43m longer than Boeing 767-200ER, is 46.4ton. The average mass of the fuselage per unit length is thus $m = 786 \text{kg/m}$. Assume this mass per unit length is the same for Boeing 767-200ER.

The fuselage consists of a system of rings and stringers attached to sheet metal. The floor separating the passenger and cargo area runs slightly below the diameter of the round fuselage. At this level of the first order engineering analysis, it is not possible to account for the individual contribution of rings, stringers, and the skin.
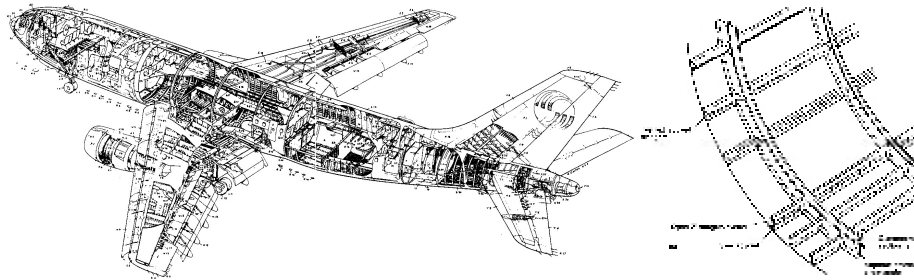

Figure 8. Internal structure of Airbus 320 (Reprinted from Ref. [25])

Instead these members are smeared into an equivalent thickness, which retains the same mass as the actual fuselage

$$p Dt_{eq} r_{Al} = m \qquad (4)$$

From the above equation, it is found that for $D = 5.03$m, the equivalent thickness is $t_{eq} = 18.4$mm.
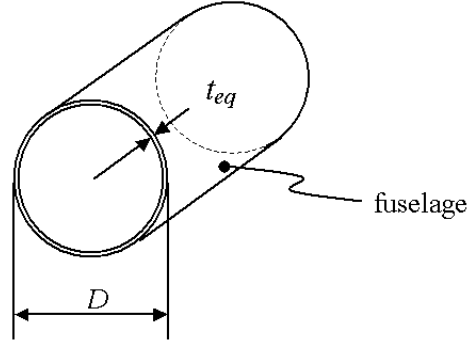

Figure 9. Simplified model of the fuselage

The building must now be characterized more exactly. The outer columns form a "fence" which can be treated as a continuous wall (see next section for the structural details). The fuselage can be assumed to crush and fold upon contact. The floor, on the other hand, is a single, relatively narrow structure of width $w_o = 0.9$m.

The quasi-static crushing of a uniform circular tube representing the fuselage has been studied by dozens of researchers including one of the present authors. Following Wierzbicki et al, the expression for the mean crushing force is, [26]

$$P_m = 7.9 s_{Al} t_{eq}^{1.5} D^{0.5} \qquad (5)$$

Taking the actual data and assuming the flow stress for aluminum alloy is $s_{Al} = 350$MPa, one gets $P_m = 15.5$MN.

Multiplying the crushing force by the total length of the fuselage, the energy absorbed by crushing of the fuselage is $E_{fuselage} = P_{fuselage} \cdot l_f = 753$MJ. It will be shown later that the actual energy is smaller.

Now the fuselage is getting engaged with one or two floors of the height $w_o = 0.9m$ each. The floor is relatively narrow compared to the diameter of the fuselage and may in fact slice through the fuselage and cut it into two or three pieces. Wierzbicki [27] derived an approximate solution for plastic resistance of a blunt object cutting into thin sheet, such as tubular wall of the fuselage model. He identified the so-called "concertina" tearing mode, which consists of two diverging cracks enclosing a strip which progressively folds back and forth. A photograph of the damaged pattern induced by a rigid punch of width w_o is shown in Figure 10.
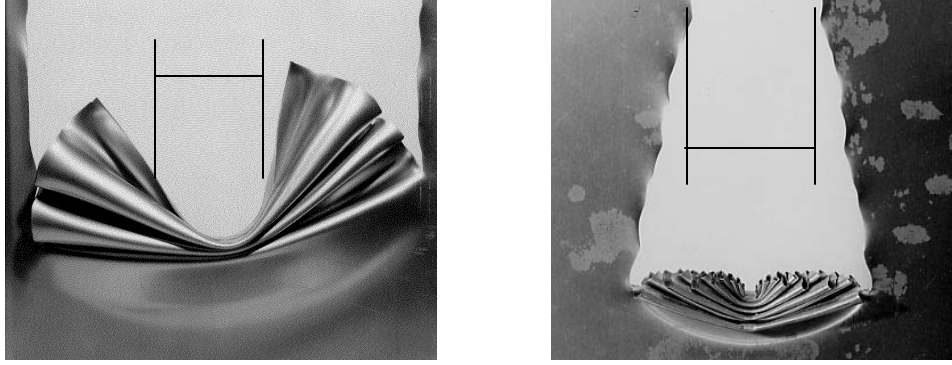
Figure 10. Concertina tearing of a sheet by a blunt object with parallel cracks (left) and diverging cracks (right)

The mean cutting force can be calculated from the equation

$$P_{cut} = 3\boldsymbol{s}_{Al} t_{eq}^{5/3} w_0^{1/3} \tag{6}$$

The total resistance of the fuselage to the cutting mode will depend on the relative orientation of the floor with respect to the fuselage cross-section. Some possible cases are depicted in Figure 7 for the North and South Towers.

In the case of contact with one floor, the cutting force is $P_{cut} = 2.6\text{MN}$. Should fuselage hit two floors at a time, the cutting force becomes $2P_{cut} = 5.2\text{MN}$.

The above forces are forces resulting from the so-called "*visible*" dissipated energy. As pointed out by Riera [20], another important contribution to the contact force comes from the momentum transfer and is given by

$$P_{momentum} = \boldsymbol{m}_f V^2 , \tag{7}$$

where $\boldsymbol{m}_f$ is mass per unit length of the cut area of the fuselage and V is the instantaneous velocity of the impacting object. It is estimated that $\boldsymbol{m}_f = 89.4\text{kg/m}$ for cutting through one floor and $\boldsymbol{m}_f = 178.8\text{kg/m}$ if the fuselage is engaged in two floors cutting. In the case of South Tower, Eq.(7) gives $P_{momentum} = 5.2\text{MN}$ for the scenarios of one floor and $P_{momentum} = 10.4\text{MN}$ for the scenarios of two floors cutting at a time. The total cutting force becomes then $P_{total} = 7.8\text{MN}$ and $P_{total} = 15.6\text{MN}$ for the cases of one or two floors respectively.

This force should be compared with the force needed by a "rigid" fuselage to cut through a deformable floor. The lower value of the two will be taken in the global energy balance calculation. This will be done in the next section. Should the cutting force of the fuselage by one or two floors will be smaller than the cutting force of the floors by the fuselage, then one can calculate the energy absorbed in the cutting mode as a product of the cutting force times the length of the fuselage. (The reaction force produced by Riera term, Eq.(7) does not contribute to the energy dissipation because the corresponding displacement is zero.) In our case the energy consumed in cutting the fuselage is equal to

$E_{fuselage/cut} = P_{cut} \cdot l_f = 127\text{MJ}$ or twice as much if the fuselage cuts through two floors. In the final energy calculation, we are using a mean value between those two estimates, which is $E_{fuselage/cut} = 190\text{MJ}$.

It should be noted as the fuselage would interact with the floor structure, it is likely that material debris had piled up at the head of the airplane, widened the contact area. Therefore it is possible that somewhere during that phase the rear portion of the aircraft will be subjected to progressive crush rather than cutting. However, switching from one failure mode to the other is highly speculative. The maximum possible value of the crushing energy is $E_{fuselage/crush} = P_m \times l_f = 753\text{MJ}$. In fact, the fuselage that has been weakened by two or three cuts will not develop its full resisting force which otherwise will be offer by an intact cylindrical tube. In the energy calculation, we will take only half of that energy which is $E_{crush/fuselage} = 376\text{MJ}$, but this assumption is highly speculative and clearly demonstrates the difficulty in the present damage analysis.

3.3 Engines and wing damage

The engines are the only components of the aircraft that can be considered approximately as rigid bodies. Their devastating power is unmatched until they encounter an object of similar weight and strength. In the experimental study in which an engine of a transport aircraft hit a thick concrete wall, the engine itself was crashed and fractured, so it was not rigid, [28]. However, in contact with less substantial members the engine could cut and plow through the various structural members of the WTC Towers until all their kinetic energy is absorbed.

Wings of modern transport aircrafts are quite complicated structures consist of open section beams, ribs and a skin reinforced by stringers. Together they form a very stiff and strong box-type section. Determination of the strength of the wing relative to the strength of the floor structure will require a detailed finite element analysis, which we believe has not been performed to date. In order to retain the needed degree of simplicity, two models were developed. In one model the wing material is lumped into single box-type beam. In the second model, the solidity ratio are determined for both the wing and the floor and then are compared.

The main structural part of the wing is the spar – a continuous beam that extends from one tip of the wing to the other. For modeling purposes, we assumed that the mass of the wings (excluding engine) was approximately $M_{wing} = 21300\text{kg}$. This mass does not include the mass of the fuel in the wing tanks. Assuming that this mass is now uniformly distributed over the whole wing span and the wing is modeled as a thin-walled square section cross-section ($c \times 4c$) with the thickness (t_{eqw}), the equivalent thickness of the wing beam can be found from the equation

$$\left(10ct_{eqw}\right)l_w\, \boldsymbol{r}_{Al} = M_{wing} \tag{8}$$

Taking an average height of the spar to be $c = 480\text{mm}$ and the span of the aircraft $l_w = 47.57\text{m}$, the equivalent thickness becomes $t_{eqw} = 34.5\text{mm}$. The wings are swept at approximately 35° so that upon impact, external columns are contacted sequentially, one by one. However, the problem of a hollow beam striking another hollow column at a right angle and a speed of 240 m/s has not been analyzed in the literature. Therefore it is not possible, at this point in time, to give any detailed account on this interaction, between the wings and outer

column, with a higher degree of accuracy than our approximate engineering analysis. The equivalent thickness of the hollow wing beam is approximately four times larger than the thickness of the exterior columns, $t_{ext} = 9.5\text{mm}$. It is therefore reasonable to treat wings as rigid bodies upon impact with exterior columns. By the same token, the equivalent thickness of wings is smaller (about half) than the equivalent thickness of the floor structure (to be calculated in the next section). Consequently it would appear that the floors will cut through the wings without being severely damaged themselves. In actuality the wings are constructed as a 3-dimensional lattice of open section beams, ribs and sheet metal skin that maybe of comparable strength to the floor trusses. However, interaction between two 3-dimensional space frames impacting each other is too difficult to carry out analytically at the present level of approximation.

In the alternative model, we are calculating the solidity ratio of both the wing and the floor defined by $\overline{r} = \dfrac{\text{Mass}}{\text{Structural Volume}}$. Note that the structure volume is meant as a volume enclosed by the outer periphery and not the material volume. Thus, for the wing $\overline{r}_{wing} = \dfrac{21.3}{0.48*(4*0.48)*47.57} = 0.49\text{ton/m}^3$ and $\overline{r}_{floor} = \dfrac{1466}{2891*0.9} = 0.56\text{ton/m}^3$. The magnitude of both solidity ratio are similar but it would appear that structure with higher solidity ratio should cut through the one with the lower solidity ratio without being damaged. According to the above model, damage of wings and floors should occur almost simultaneously. No relative level of crush resistant can be calculated, but the energy approach will still be valid.

It can be conjectured that those portion of the wing that fit in-between the floors will penetrate all the way to the core columns and will be broken by the core columns, which are much stronger. From the comparison of the airplane with the floor plan of the South Tower, shown in [15], it appears that the wing encounters the first row of six core columns. The wing beam will most probably fail by the shear mode to be described in Section 5 or simply by crushing. Assuming a crushing mode to be more realistic, the energy absorbed during that process is equal to $E_{wing} = 4p M_{ow} l_{wc} = 20\text{MJ}$, where M_{ow} is the full plastic bending moment of the wall of the wing box and $l_{wc} = 15\text{m}$ is the estimated length of the wing that fit in-between the floors and subsequent impact the core structure. Current research is underway to determine the accuracy of this approximation.

The other part of the wing that will in touch with the floor structure would probably be fragmented into smaller pieces. It is not clear what is happening next with this already disintegrated wing structure. There are approximately 25 columns on the way of this debris. The process of cutting would have slow down the wing velocity, which have already being diminished by the earlier contact with the exterior columns and floors. The created debris will impinge into the core columns causing them to bend and stretch but not necessarily fracture. While the corresponding analysis is presented in the next section, it is impossible to make any statements about the degree to which wing structures will be subjected to further fragmentation.

In our best estimate, the plastic and fracture energy absorbed by disintegrating the airplane can be summarized as follows

Energy to crush the fuselage	$E_{fuselage/crush} = 376\text{MJ}$
Energy to of cutting the fuselage	$E_{fuselage/cut} = 190\text{MJ}$
Energy of breakup of wing(s)	$E_{wing} = 20\text{MJ}$
Total energy absorbed by airplane	$E_{airplane} = 586\text{MJ}$

4. Building failure

4.1 Design, prefabrication and construction

This section will give an overview only of those structural aspects of WTC Towers that are relevant to the subsequent failure analysis. In order for the two buildings to withstand the tremendous wind loads faced by a structure of such unprecedented height, the double "tube building" model was employed. The name "tube model" comes from the building being shaped like a stiff "hollow tube" of closely spaced columns on the exterior, and floor trusses which extend across to a central core on the interior. This shape allows the building not only to withstand wind loads, but 'reportedly' also a collision with a large commercial airplane flying at lower speed. The validity of the latter claim is questioned in this article. The vertical steel and concrete core that forms the center of the "tube" supports approximately 60% of the total gravity load of the building, while the outside shell bears the remaining 40%. The towers were built very modularly and consisted of many prefabricated pieces, such as exterior panels, floor trusses etc. On the other hand, the core structures were constructed more traditionally in the "cage" type design.

4.2 Exterior columns

The 64m (208 ft) wide façade is, in effect, a prefabricated steel lattice. The exterior columns are narrowly spaced and finished with a silver-colored aluminum cladding. The main building block of the outer structure was a prefabricated element, which was comprised of 3 floors, was 11 m high and 3.07m wide, Figure 11.
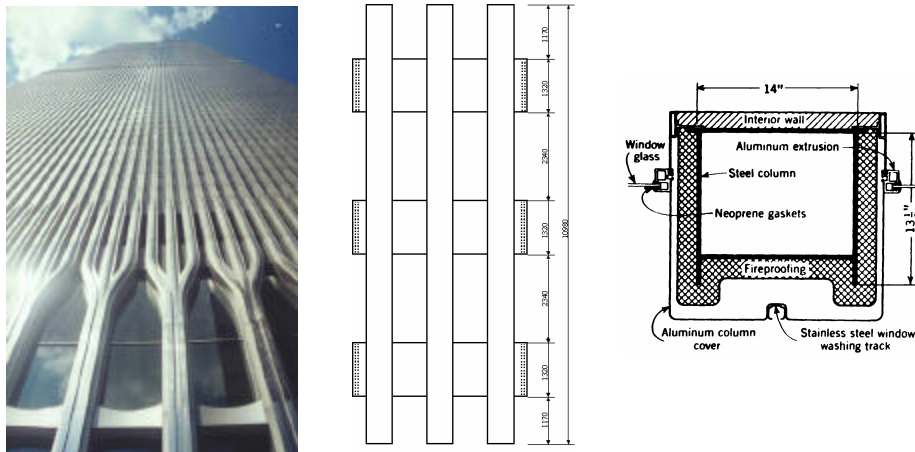

Figure 11. Prefabricated panel consisting of three columns of three-floor-high

The prefabricated panel consisted of three columns connected by 3 transverse plates, called spandrels. The steel columns are of square cross-section ($b \times b \times t = 356\text{mm} \times 356\text{mm} \times 9.5\text{mm}$). and they were spaced 570 mm apart from each other. The segments were staggered and bolted to their neighboring elements in every direction, Figure 12.
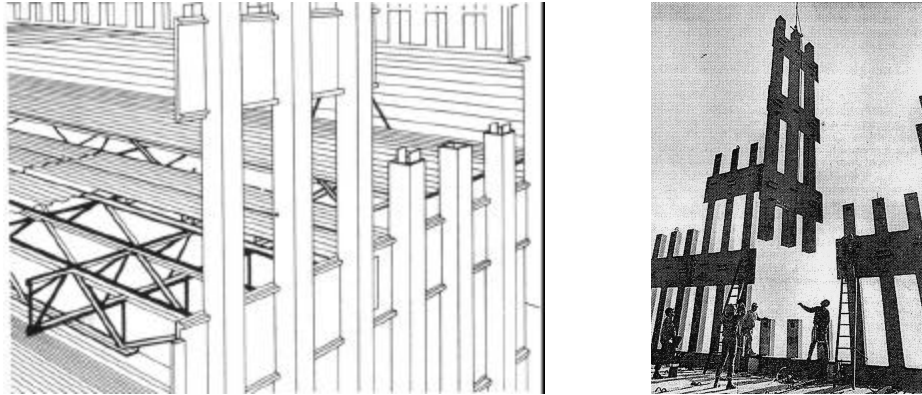

Figure 12. Assembly of the external wall units (alternately staggered in one-story heights) and floor units.

Each column was a box structure, almost square, with a assumed wall thickness of $t_{ext} = 9.5\text{mm}$. In actuality, the exterior columns were variable in thickness of 12.5mm at the bottom of the buildings to 7mm at the top. The true columns thickness of that portion that was hit is not known to the authors. In the present analysis, the columns were assumed to be made of the medium grade A36, constructional steel characterized by:

Yield Stress:	$\boldsymbol{s}_y = 250\text{MPa}$	
Ultimate Strength:	$\boldsymbol{s}_u = 475\text{MPa}$	
Elongation (Fracture Strain):	$\boldsymbol{e}_f = 0.23$	

The so-called energy equivalent flow stress, calculated from the above values, and using the power-law approximation of the stress strain curve, is $\boldsymbol{s}_{A36} = 396\text{MPa}$.

4.3 Floor Structure

In addition to carrying the normal vertical loads, the floor system had to act as a diaphragm to stiffen the outside wall against lateral buckling forces from wind load pressures, and had to be very strong. Thus, in order to maintain some level of cost and weight efficiency, they were quite complex. The floor construction was of prefabricated trussed steel, 800 mm (33 in) in depth that spanned the full distance to the core. There was a primary truss system, which supported a corrugated steel plate on which was poured a 100 mm thick, lightweight concrete slab. The author did not have access to the technical drawings for each Tower. However, dimensions of many key structural members can be retrieved from generally available information, such as the total weight of the floors. The total weight of each floor is M_{floor}=2200 tons and the office floor area was A_{office}=2891 m^2. Subtracting from the total floor weight the weight of the concrete slab of 734 tons, the weight of structural steel in each floor is calculated to be 1466 tons. The above calculated data will be used to form estimates of the energy absorbed by the floor structure.

Figure 13. Relative size of core column compared to the exterior column. We have estimated the geometry of the core column from this photograph.

4.4 Core columns

Inside each tower there were 44 large, concrete reinforced, steel columns, which enclosed elevators, stairways, and utility space. Again, the author's inquiries to ascertain exact values for the core column dimensions failed. However, one is able to estimate these values by comparing the size of core columns to the size of exterior columns as captured in photographs of the site, such as the one shown below. With an accuracy compromised by the poor resolution of the photographs available, we determined that each column had a thickness of 67mm, and dimensions of $950\text{mm} \times 312\text{mm}$ in rectangular cross section. It is not certain if all core columns shared identical cross section, but our calculations could easily be revisited when more precise data on their exact geometry becomes available. It is hoped that we will be able to eventually retrieve exact dimensions of core column in the course of our continuing research.

4.5 Connections

Each prefabricated panel was bolted through spandrels to its horizontal neighbor with 2 rows of 18 bolts each. This is, again, an estimated value, but as you will see later on in this discussion, a bolted connection is so weak that the diameter of these bolts within plus and minus 5mm is really insignificant. It is easy to calculate the cross sectional shear strength of the bolts, and is approximately half of the shear strength of the parent material, and possibly less because of stress concentrations. The photographic coverage of "Ground Zero" has proven that individual, prefabricated panels were almost all separated at these bolted seems, and it can further be said that it was actually the bolts which fractured rather than the material in the spaces in-between them. Concerning the connection between the staggered, prefabricated elements in the vertical direction, there were only four bolts adhering the interfaces of two columns. The bolt cross sectional areas in these joints comprised approximately 2.3% of the column cross-section. Clearly there is a gross incompatibility between the strength of the connections (in shear and in tension) with the strength of the columns themselves. Elementary, beam-bending theory calculations show that these bolts would have failed with only 1 mm transverse deflection of the columns (loaded as a beam). For all practical purposes they may be assumed to have negligible strength in bending, shear and tension. The strength of connection between the exterior wall and floor trusses is discussed in Section 6.

5. Damage estimate

5.1 Failure of exterior columns

The overall picture of the damage to the exterior shell is shown clearly in Figure 5. An interesting overlay of the outline of the plane on the North Towers impacted face is shown in Figure 14. One can clearly distinguish the fuselage together with the vertical and horizontal fins of the tail section, as well as two smaller holes driven by the engines. In the FEMA/ASCE report, it was estimated that the length of the damage area was approximately 31m, which is shorter than the wing span which is 47.57m. Therefore, it can be concluded that the extreme portion of the wings didn't cut through the columns but is actually deflected themselves. The damage extended over five floors, which is easily to see by counting rows of detached aluminum cladding, each one story high. From Figure 15(a) and 15(b) one can see that 33 and 23 rows of columns were cut by the impacting aircraft to the North and South Towers respectively.

Let us look at the exterior columns individually. The plane could have struck the building with its nose localized at the point of a floor junction; this would have been the strongest resisting point. It could have struck where two of the steel lattices had been joined together via steel bolts; this being the weakest of the defenses. Or, it could have struck simply in the middle of the beam sections between floor junctions.
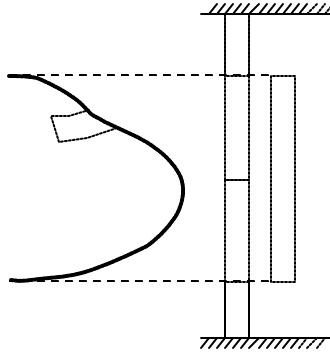

Figure 14. The shearing failure of exterior columns by the aircraft fuselage
(and wings). Details given in Figure 25.

Additionally, the impacted members were continuously supported by their own lateral inertia which is proportional to the mass per unit length and the acceleration $m\ddot{w}$. The latter effect was, in fact, the decisive type of for this range of craft velocities. Most, if not all, damaged columns seen in Figure 19 exhibited a clear 'cut' produced by shear failure.

The instantaneous plastic shear force developed in the cross-section is $\dfrac{\boldsymbol{s}_0}{\sqrt{3}}4bt$. Upon complete separation, the plastic energy dissipation is obtained by multiplying the shear force by the thickness of the sheared –off element. Thus, the upper bound on the shear energy per one cut is

$$E_{cut} = \frac{\boldsymbol{s}_{A36}}{\sqrt{3}}\left[\left(2bt_{ext}\right)t_{ext} + \left(2bt_{ext}\right)b\right] = \frac{2\boldsymbol{s}_{A36}}{\sqrt{3}}bt_{ext}\left(t_{ext}+b\right) \approx \frac{2\boldsymbol{s}_{A36}}{\sqrt{3}}b^2 t_{ext} \tag{9}$$
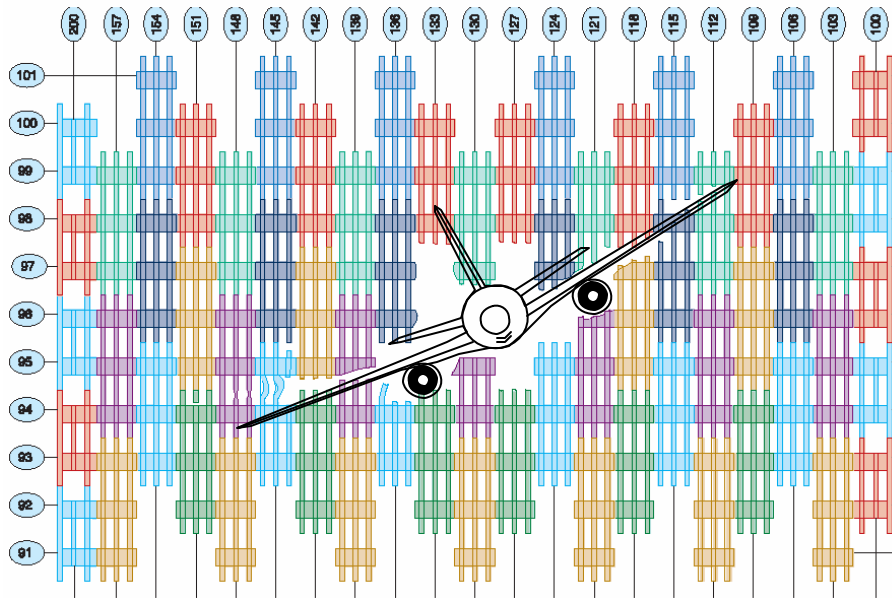
Figure 15(a). The outline of the airplane superposed on the hole driven in the
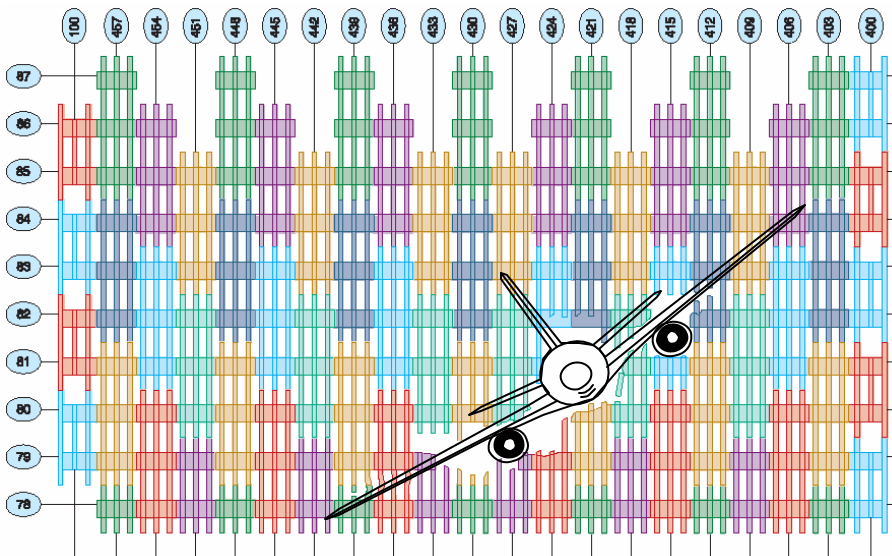exterior wall of the North Tower

Figure 15(b). The outline of the airplane superposed on the hole driven in the
exterior wall of the South Tower

The alternative failure mode is plastic bending of the cantilever beam but it is very
unlike that this failure mode would occur under high velocity impact because it will require the
beam inertia to be activated. Recent results of the numerical study have conclusively proven
that exterior columns fail by the shear type of failure, [2], see also Figure 25.

Multiplying the energy per column (Eq. 9) by the number of damaged columns the
total energy dissipated by the external columns of the South Tower is

$$E_{external\_column} = \left(\frac{2s_{ext}}{\sqrt{3}} b^2 t_{ext} \right) 2 \cdot 23 = 20\text{MJ} \qquad (10)$$

This is only a small fraction of the available kinetic energy of the aircraft.

It is recognized that there is a momentum transfer during the cutting process and additional energy is lost during that process. Teng and Wierzbicki [2] estimated that this additional energy loss is approximately $\Delta E = E_{kinetic,wing} \dfrac{M_{column}}{M_{coumn} + M_{wing}}$ where M_{column} and M_{wing} denote the respective masses of the columns and the wings that are in contact. According to the calculation performed by Teng and Wierzbicki [2] the mass ratio is 0.0783, which means 7.83% of the initial kinetic energy of the wings (96MJ or 2.6% of the total initial kinetic energy) is lost in cutting the exterior columns. What can be concluded with full confidence is that the plastic work used for fracturing the top and bottom of flanges as well as two webs is significantly smaller than the kinetic energy lost during the process of momentum transfer.

5.2 Failure of floors

Now that the plane has made it through the exterior membrane of the tower, the floors present the next opportunity to dissipate its remaining kinetic energy. How many floors did the plane collide with? How much energy does it take to move the airplane through the entire 10.7m all the way to the core? How can we model them? Our analysis uses several engineering approximations to effectively analyze three different models of the floor destruction.
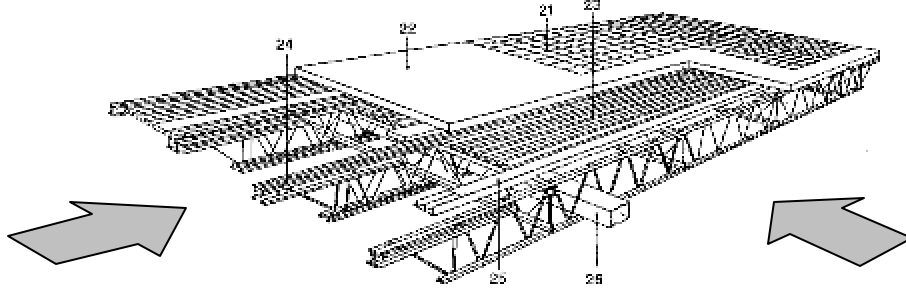

Figure 16. Aircraft impact direction with respect to the layout of floor structure

The complexity of the floor structure, as confirmed by the figure above makes the analysis very difficult. The floor structure can essentially be regarded as a longitudinally stiffened plate. Paik and Wierzbicki [29] and Braco and Wierzbicki [30] showed that a good engineering approximation for calculating resistance of such plates to crushing and cutting forces is obtained by the so called "smearing technique". In this technique the evenly spaced steel trusses are condensed into an equivalent thickness of uniform plate. Dividing the total volume of the steel imbedded in each floor by the floor area, the equivalent thickness becomes

$$t_{eq} = \frac{M_{steel}}{r_{A36} A_{floor}} = 65\text{mm} . \qquad (11)$$

Various cutting and tearing failure modes of plates were identified and studied in the Impact and Crashworthiness Laboratory at MIT in conjunction with the project on grounding

damaged of oil takers and ships. Photographs of two typical failure modes are shown in Figure 17.
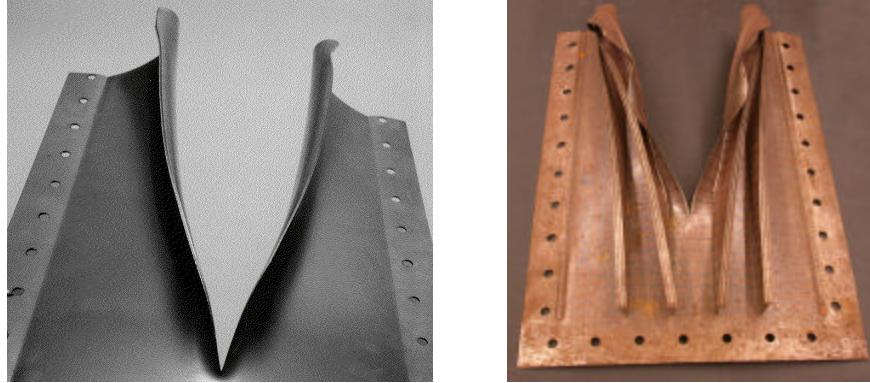

Figure 17. Unstiffened and stiffened cut by a shape wedge

The pure cutting mode shown above involves one running crack followed by curling and stretching of flaps. Note that the stiffeners, if any, are deforming and curling together with the plate. The picture of damage of longitudinally stiffened plate, shown on the right could correspond more closely to the failure of the WTC tower floors in which floor trusses could be considered as stiffeners. Because this mode can only be activated by a sharp wedge, unlike a blunt fuselage, it will not be pursued any further.

It is believed that the more applicable failure of floors could be the so called "concertina folding" mode, see Figure 10. The concertina mode can in fact be initiated by any blunt object such as the aircraft fuselage or wing. The failure mode consists of two parallel or diverging cracks with the plate folding back and forth between the cracks. The material is essentially piling up in front but this is not affecting the structural resistance. The solution to this rather complex problem involving combined plastic flow and fracture was given by Wierzbicki, [27]. Using realistic assumptions, he derived a very simple expression for the resisting force

$$F_{floor/fuselage} = 3\boldsymbol{s}_{A36}t_{eq}^{5/3}D^{1/3} \qquad (12)$$

where D is the width of the cut, the equivalent thickness of the floor is $t_{eq} = 65\text{mm}$ and the flow stress of A36 steel is $\boldsymbol{s}_{A36} = 396MPa$. It can be shown that the force for the fuselage for the 5.03m diameter to cut through the floor is much higher than the force of the floor to slice through the fuselage. That leaves only the wings and engines as airplane member that are sufficiently strong to cut through the floor.

The diameter of the engines is approximately $D_{eng} = 3m$ while the height of the wing modeled as a single beam is $c = 480mm$. The corresponding resisting forces are

$$F_{floor/engine} = 3\boldsymbol{s}_{A36}t_{eq}^{5/3}D_{eng}^{1/3} = 18\text{MN} \qquad (13)$$

$$F_{floor/wing} = 3\boldsymbol{s}_{A36}t_{eq}^{5/3}c^{1/3} = 10\text{MN} . \qquad (14)$$

The floor span of the South Tower impacted side is 10.7m, however for a diagonal impact, see Figure 4, the length of the damaged floors will be much larger. It is observed that the left wing traveled some 10m, while the fuselage and right wing traveled an average of 50m. Therefore, taking an average between those two, the length of the damaged floor is taken to be

equal $l_{floor} = 30m$ (an estimated average length of 18.3m for the North Tower), the total energy dissipated on destroying the floors are

$$E_{floor} = \left(2F_{floor/engine} + F_{floor/wing} \right) \cdot l_{floor} = 491\text{MJ} .$$ (15)

In the above equation the contribution of only one engine was taken into account, because the other one (or at least part of it) felt a few blocks from "Ground Zero" meaning that the engine did not engaged with the floors. For the North Tower, it also assumed that only one engine is engaged in impacting with floor. The above estimate includes only the energy in the plastic deformation and fracture but does not take into account the energy loss on the momentum transfer. The problem of simultaneous energy dissipation and momentum conservation was recently solved by Teng and Wierzbicki, [15]. According to their calculations , the additional loss of kinetic energy is proportional to the ratio of the impacted mass to the sum of impacted and impacting mass. Thus, the results of the present calculation would very much depend on the magnitude of the floor mass that was accelerated by the impacting airplane. It is a very difficult task because there is no clear indication how many floors were engaged in the contact with airplane and which part of the airplane was able to fit in-between the floors. Our best estimate is that some 15% of the initial kinetic energy was lost on pushing the floors. This additional energy loss is then $\Delta E_{floor} = 0.15* E_{kinetic} = 549\text{MJ}$. This point will be revisited should more precise information becomes available for full-scale simulation. In summary, our best estimate on the energy loss for the damaging of the floors themselves is 1040MJ for the South Tower.

Once again, a word of caution should be added here regarding the accuracy of our mass estimate. At the present level of modeling, it was difficult to assign a unique mass of wing as well as to tell what part of the mass of the affected floors have been in contact with fuselage and wings and will accelerated during the impact event. Only a detailed finite element modeling and calculation will give definite answer to this question. Such a project is under development with possible sponsorship of NIST.

5.3 Failure of core columns

The core columns are much stronger than the exterior ones. The response of a plastic beam, loaded dynamically, occurs usually in three phases dominated respectively by shear, bending, and membrane action, Jones [31], Hoo Fatt [32]. It is assumed that by the time the core structure is reached the impacting debris of the aircraft will have been slowed by exterior columns and floors and would also have been broken down even further so that the loading induced on the core columns was distributed rather than concentrated. Under those conditions, the most probably failure mode would not be shear, as was the case with the exterior columns, but rather bending and, or membrane types of failure.

We do not have complete information on the manner in which the core columns were joined. Therefore, in order to complete this analysis, two different models could be employed. The first model, will apply to the weakly joined case, such as a single-pass weld on the thick-walled (67mm) beam. Such a joint would be easily broken and, similarly, as in the case of the exterior columns, the core columns can be treated as two cantilevered beams at fixed distances to the floors. However, the global bending mode of the core column will entail global inertia of the beam which, we think, should be excluded because of the short duration of the impact phenomenon. Therefore the bending deformation mode will not be pursued any further. In the second model it will be assumed that the connections have the same strength as a cross section of the parent material. In this case, the membrane deformation mode is appropriate.

The plastic energy required to stretch the core column in the membrane mode all the way to fracture is:

$$E_m = Al_m \boldsymbol{s}_{A36} \boldsymbol{e}_f \tag{16}$$

where, $A = ab - (a-2t)(b-2t)$, is the cross sectional area, and l_m is the length of the column. Taking l_m equal to the length of one, two or three floors, the membrane energy is listed in Table 2.

Because the core columns are so strong and dissipated so much energy, assumption about the effective cross-section area and the length of the damaged column will have a decisive effect on the number of damaged columns. It is here that information from the crash site about the mode in which core column failed would be extremely helpful. In the absence of the above data, we must consider six different cases in that table below.

Table 2. Six different cases damage of a single core column

Dissipated energy (MJ)	1 floor		2 floors		3 floors	
	South	North	South	North	South	North
Membrane only (strong weld)	51	51	102	102	153	153

It should be noted that not all impacted core columns will be deformed and fractured. That could be the case that only a few columns while other core columns could be subjected to certain degree of bending and stretching without fracture. A devastating effect of this type of deformation on the overall survivability will be explained in the next section.

5.4 Energy balance

We are now at a sufficient point to return to the global energy balance (see Eq. 1) which can now be solved for E_{core}.

$$E_{core} = E_{kinetic} - \left(E_{plane} + E_{external\_column} + E_{floor} \right) \tag{18}$$

The energy required to damage the exterior columns, the floors, and the aircraft itself has already been estimated. Also, we know the total energy introduced to the Tower. So, the only unknown is the total energy absorbed by the core. We can now graphically illustrate the break-down of energy dissipation in this impact.
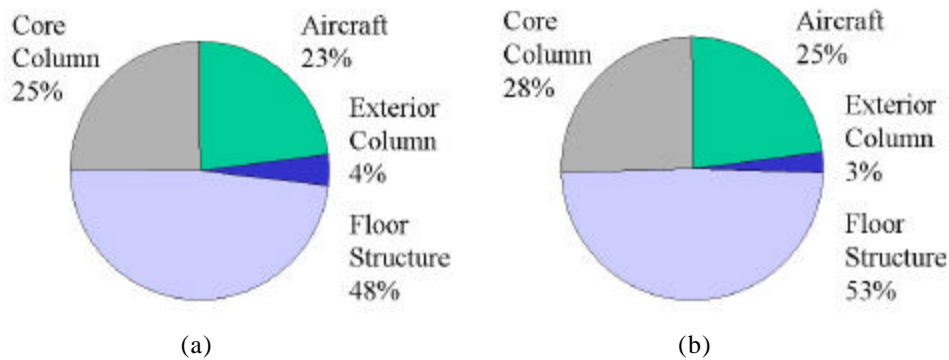

Figure 18. The contribution of various members to the energy dissipated during the initial impact. North Tower (a), South Tower (b).

According to our best estimate, the core columns absorbed $E_{core} = 1025MJ$, which is 52% of the total kinetic energy introduced by the aircraft. The total number of destroyed core columns is a ratio of the total energy available - core energy E_{core} to the amount of energy required to fail a single core column.

Depending which case considered in Table 2 will be valid, **the number of destroyed core columns in South Tower will vary between minimum of 7 and maximum of 20**. It should be noted that the prediction for the North Tower would be different for two reasons. First, the impact velocity is smaller and hence the kinetic energy induced by the airplane is less. Second, the airplane impacted the tower on different side correlating with the core structure orientation, so that the energy dissipated by these longer floors was larger. Taking the each of the factors above into consideration, **the predicted number of damaged core columns in the North Tower will vary between 4 and 12.** There will be an enormous difference between the ways in which the global collapse was initiated in both towers. Effect of the local damage on the global collapse of each tower is discussed next.

6. Comments on structural collapse

Until this point, the focus of this article has been the instantaneous damage incurred by the aircraft impact, which was localized within few floors of each tower. Yet, at the same time, the initial impact set the stage for the complex series of structural weakening and failures that finally led to a complete collapse of both towers. The manner in which these two stages of failure are related is the subject of extensive debate.

The following section in not intended to perform a full analysis of the global collapse but rather bring up few important issues relevant to the accident reconstruction. Two distinguishable schools of thought have emerged from such debate. These are

- Fire Dominated Theory
- Impact Dominated Theory

The first of these theories requires that prolonged, ultra high-temperature fire degraded the steel to such a point as to induce progressive failure from such a weakened state. By contrast, the second theory, which has been strongly supported by the analysis brought forth in this article, requires that the initial aircraft impact brought the building to the verge of instability. So close to this point, in fact, that only a small shift in loading or a minute decrease in structural strength would have resulted in the catastrophic collapse. In a brief discussion below, each of these theories is described in more detail.

6.1 Fire dominated collapse theory

While the majority of the paper only dealt with the instantaneous damage introduced by the aircraft impact, the effects produced by the secondary damage incurred by the fire deserve careful consideration. One cannot deny that the situation became much more serious on a structural level when energy was introduced in the form of burning jet fuel. The general idea is that the heat gradually affected the behavior of the remaining material after the impact, thus decreasing its elastic modulus, yield stress and increasing the deflections. This subject has been extensively covered via mass media, and one of the most important aspects of this argument is the observation that whatever fire protection the steel was prepared with, was shaken lose by the impact and thus unable to perform as designed. A jet-fueled fire is not what normal office fires are like and thus the safety systems may have been overcome considerably faster than expected. Our analysis does not deny these heat-induced contributions to the collapse, rather we fully agree that the fire effects played a large role in the deferred damage.

Yet, we do believe that the primary damage suffered by the South Tower via the initial impact alone was severe enough to bring it down with very little outside help. This is the point of view that has been given almost no attention or thought. At the same time, several arguments are introduced later in this article that support the theory that the North Tower collapse was facilitated by fire.

6.2 Initial damage dominated collapse

With respect to the impact dominated theory, the following issues, when assimilated into a cohesive failure theory, form this argument:

- Effect of Stress Concentration
- Initial Extent of Damage: as measured by the number of destroyed floors and columns
- The location of the damaged zone with respect to the axis of symmetry of the structural cross-sections
- The redundancy of the structural systems
- The safety factors which particular zones of the towers were designed for

We now proceed with a sequential discussion of the factors listed above.

Effect of stress concentration. The exterior column on each of the four sides of the building carry a uniform load in the vertical direction. This load increases from top down due to gravity. Consider a "control" section of several floors of the height l_o. The outer "facade" of a tower can be modeled as a plate strip under uniform compression. The so-called far field stress due to the weight of the portion of the building above is denoted by s .
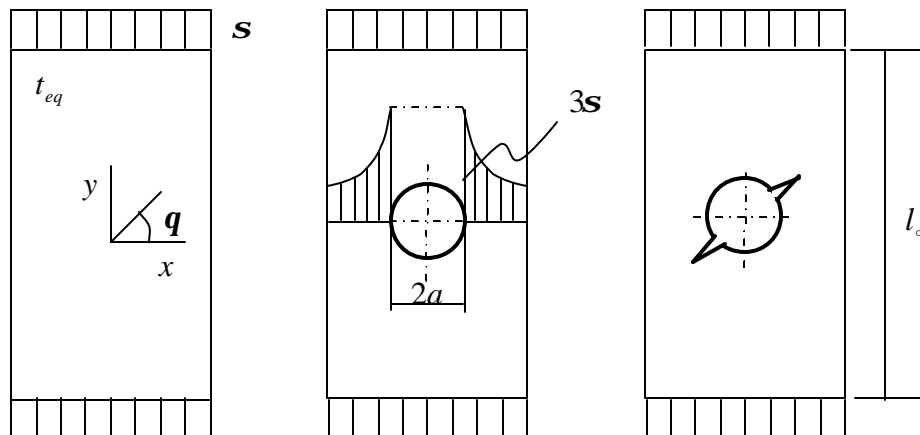

Figure 19. Stress concentration around a circular hole in a plate. Note that there is a stress singularity at the tip of any crack emanating from the hole.

Now imagine that a hole of radius a is driven into the center of the plate. The in-plane compressive stresses will be redistributed and will be concentrated near the hole. The exact elastic solution of this problem was worked out by Timoshenko [33]. The vertical component of the stress s_{yy} varies along x-axis according to

$$s_{yy} = s\left(1 + \frac{1}{2}\frac{a^2}{x^2} + \frac{3}{2}\frac{a^4}{x^2}\right) \tag{19}$$

Denoting the maximum stress at the edge of the hole $x=0$ by $\left(s_{yy}\right)_{max}$, the stress concentration factor becomes $g = \left(s_{yy}\right)_{max}/s = 3$. This suggests that the exterior columns adjacent to the hole could yield (or buckle) if the safety factor is less than three. We have extended the above result to the more general case of linearly variable compressive load due to gravity. The stress concentration now depends on the $\frac{l}{l_o}$ ratio (refer to Figure 20.)

$$g = \frac{3 + 1.65\dfrac{l}{2l_o}}{1 + \dfrac{l}{2l_o}} \tag{20}$$

Taking a realistic value $\frac{l}{l_o} = 0.1$, the stress concentration factor reduces from 3 to 2.93. Thus, the variable gravity load will only reduce the stress concentration factor as compared to uniform load.

The hole driven in the outer facade by the airplane is not circular as smooth. In the next level of approximation it can be modeled by a circle with two symmetric cracks representing narrow cuts made by tips of the wings. Elastic fracture mechanics tell us that the stress concentration factor is infinitely larger at the crack tip but decays rapidly to a constant far-field value.

So why did the columns adjacent to the sharp edges of the hole not collapse instantaneously? This is because the assumption of the plane stress solutions are not satisfied by the grillage. The formulation of plane stress (thin plate) elastic problem requires that shear stresses be transmitted from section to section. This assumption is not met by the grillage-type external structure of the WTC Towers. The shear stiffness and strength of transverse plate strips welded to much heavier columns and bolted to adjacent, prefabricated sections is much smaller than the stiffness in the vertical direction. Therefore, local weakening in the form of a hole may not produce local stress concentration but rather more global redistribution of forces. We will therefore explore another limiting case in which shear resistance is removed altogether and the outer facade is assumed to be composed of a system of individual columns.

A limited modeling of the residual strength of the damaged façade was performed for the FEMA study (Chapter 2 in Ref.[1]). It was found that the safety factor of the exterior columns was an order of 5. However, this safety factor was reduced to unity for column at the edge of the aircraft produced hole. Therefore, the present closed-form solution is in fully agreement with the linear numerical analysis.
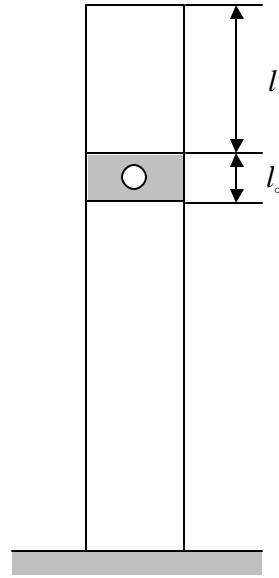
Figure 20. Stresses due to the dead weight will be concentrated around the hole

Initial extent of damage: The derivations of internal damage were taken purely from energy considerations, and thus, yielded only scalar representations of such damage expressed by the quantities of damaged floors and columns. For example, the number of damaged core columns, which bear approximately 60% of the entire gravity load of the building, was determined, in the previous section, to be 7 to 20 for the South tower. As the total number of core columns that existed was 44, these quantities represent more than 16% to 45% of the total core strength, respectively. Thus, is it correct to say that the remaining columns and load bearing members were immediately overloaded by a factor of 1.2 to 2.5? Well, this depends on the vectorial character of the impact and the zone which was effected. This brings us to the next issue.

Location of damaged zone: From the trajectory of the aircraft impacting the South Tower described in Figure 4. it is clear that the impacts of aircraft were not symmetric with respect to the centroids of the tower's cross-section. Both the outside columns and the inner columns were destroyed in asymmetric manners, and thus the locations of the centroid of the cross-section was shifted considerably. (See Figure 22 center) Therefore, an overturning moment, due to the gravity load, was immediately created, leading to non-uniform distribution of the load over the core and peripheral columns.

This situation is explained by a very simple, one-dimensional model of a mechanical system consisting of 3 columns, refer to Figure 21. In the intact state, the three columns are bearing equal loads of $W/3$ each. Two cases will be considered; one in which one of the member was entirely cut and the other one in which the same member is severely bent. If we remove F_2, that is weaken the structure symmetrically, then the load above it, W, uniformly redistributes itself and from force equilibrium, F_1 and F_3 are bearing equal loads of $W/2$ each, at the same time, moment equilibrium is satisfied identically. However, if we weaken the structure in an asymmetric manner, that is, remove F_1, then the force and moment equilibrium yield the following equations.

$$W + F_2 + F_3 = 0 \tag{21}$$

$$F_2 H = WH \tag{22}$$
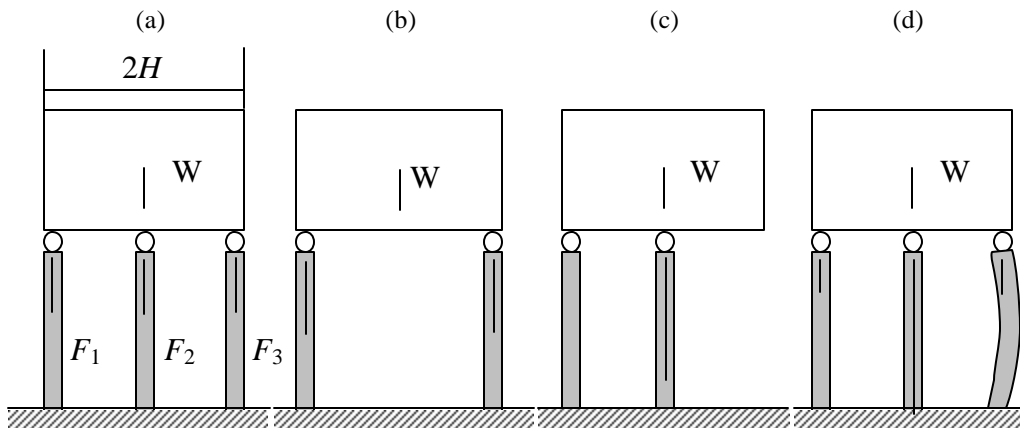
where $2H$ is the width of the model.

Figure 21. Simplified model of damaged eccentrically loaded system of column. Intact condition (a), center column removed (b), edge column removed (c) and edge column bent (d).

The solution of the above system is $F_2 = W$ and $F_3 = 0$. This asymmetric loading situation yields an inactive load bearing section opposite of the missing columns while the central columns bear the entire load. The implication of this simple observation is that before damage the loading on each column was W/3. The symmetric damage causes the load to redistribute itself as W/2 (1.5 times its original load). Where as, the asymmetric damage causes the central column to bear the entire load, W (three times its original load). Now, we were trying to solve the second case in which the peripheral column is severely dented and bent rather than being cut. In this case, the column developed fully plastic tensile force $N = s_y A$, where A is the cross-section area of the core column. A new term will then appear in the force and moment equation above and the solution of this system is $F_1 = N$ (in tension), $F_2 = -W - 2N$ and $F_3 = N$. It can be concluded that denting of peripheral core columns will cause additional increase in the overload of the centrally located core columns.

This example is relatively concrete and holds regardless of initial assumptions. It can be generalized to fit most ideally to the realistic, 3-dimensional conditions of the impact zone. The above analysis brings us to one of the most important and interesting points of this entire article, that even though only 1/3 of the interior columns in each tower may have been destroyed, in fact, 2/3 of them were rendered inactive for bearing the dead-load above.

This example is easily generalized to encompass the actual conditions that existed in the WTC accident. A conceptual picture showing the area of active and inactive columns is shown in Figure 22. Following this generalization it is possible to graphically illustrate the location of the damaged, inactive and remaining, load-bearing columns (the shaded portions of the Figure 22) at the impact zone. In fact a photographic coverage of the onset of the global collapse (Figure 23) proves the upper part of the building tilted diagonally and felt on the low part.
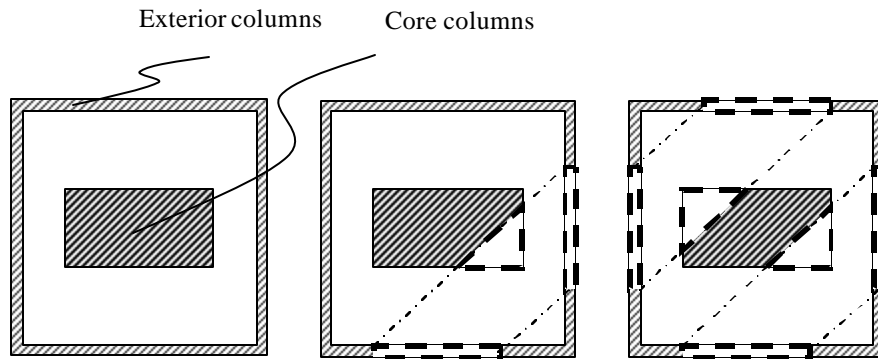

Figure 22. Conceptual sketch of the cross-section of the tower showing vertical members (left). Asymmetric damage (South Tower) removes a portion of exterior and core columns (center). Columns at mirror reflection becomes immediately inactive (right).

Figure 23. Few first seconds into the global destruction of the South Tower

Redundancy: Finally, it would be interesting to determine to what extent structural redundancy would diminish the effects of the centralized overstressing condition, which develops from asymmetric damage. The redundancy is the ability of a structure to redistribute loads around the damaged area so that one missing component will not cause global collapse of the entire system. Several lessons learned from accidents with bridges and offshore structures[1] have led to robust design of man-made structures within a large degree of redundancy.

In the case of the WTC Towers, the exact redundancy analysis would necessitate construction of a complex three-dimensional model of inner and outer tubes with continuing columns and bracing floor. Such an analysis should be performed by individuals or teams in possession of detailed structural models.

A "unique" feature of the design of the Towers was that floors were hinge-supported to the exterior columns and core structures, [1]. At the same time, shear and tensile strength of this joint was inadequate, probably an order of magnitude smaller than the local strength of members being joined.

A dramatic proof of the above statement is offered by the photograph showing large sections of the exterior wall in a free fall. No residual elements of floor truss structure could be seen attached to these sections.

What would happen if the pin-support were replaced by a built-in (welded) joint (moment connection)? An elementary beam analysis tells us that the stiffness and elastic deflection of floor beams, loaded by their own weight, be reduced by a factor of two or more. Thus, adding structural redundancy by changing the method of floor truss support will reduce or delay sagging of floor caused by fire.

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[1]   The Alexander Killian rig underwent a progressive collapse originated from just one failed member, causing 77 deaths.

Figure 24. Prefabricated columns of the exterior wall falling to the ground
completely detached from the floor structure.

What would happen if the tensile and shear strength of joints were increased by a factor of two, four, or ten? Then, the floors will keep effectively bracing inner and outer tubes, increasing the buckling strength of exterior and interior columns. Can our analysis tell what happened first: sagging of floors, which led to the detachment of floors from columns causing them to buckle, or buckling of columns causing floors to detach and fall onto one another. We think that either can be true. The impact and, thus, the damage to the North Tower were symmetric. Also, the number of destroyed core columns was fewer. It would then appear that because there was no tilting of the building, the catastrophic collapse was initiated by each floor falling into the next. This scenario would require a more prolonged effect of fire to weaken the floor trusses, which was indeed the case. The North Tower survived the initial impact for 50 minutes longer than the South Tower and then imploded.

6. Conclusions

The analysis presented in this article has quantified the amount of damage to the main structural members of the World Trade Center Towers. These numbers have been generated with the warning that they are based on assumptions and models, which had to be made because of the vast lack of exact facts, dimensions, and general calculation methods for this class of problem. There was a lack of data in two main areas. One is the plastic deformation, structural damage, crack initiation and fracture propagation in the problem of a high velocity collision of two thin-walled structures with comparable mass and strength. Research recently completed in the Impact and Crashworthiness Lab at MIT has already clarified some important issues [2, 13-15]. A sample of interesting numerical analysis of a rigid wing cutting through plastic deforming and fracturing of exterior column. The above analytical and numerical solutions are currently available as technical reports of the Impact and Crashworthiness Lab. Publication in professional journals will follow soon. The second difficulty, which arose with

respect to a lack of data, was overcome by the generality of our analysis, in which closed-form solution were derived for an entire class of structures, without the necessity of substituting exact geometric and material properties. A great deal of effort was put into retrieving the most accurate set of data available to the general public. As soon as more precise information on cross-sectional shapes and dimensions, joining metals (strengths of weldments) etc. become available, we will be able to quickly reevaluate our calculations and introduce corrections to our results.
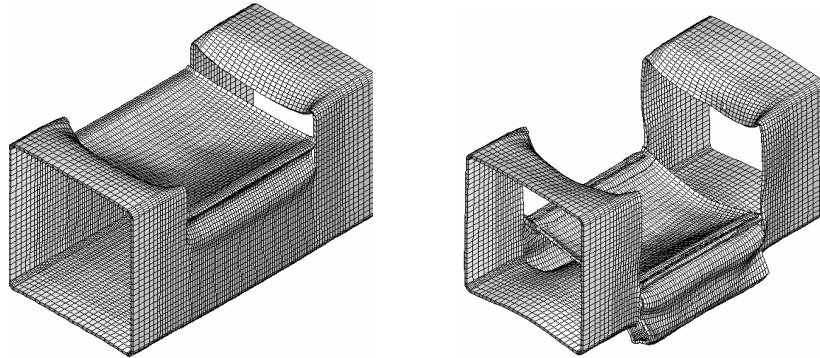

Figure 25. The problem of rigid mass (representing airplane wing) cutting through the exterior column has been solved numerically by Zheng and Wierzbicki [13].

While the extent of damage to the exterior is clearly visible, and the number of damaged floors is also easily estimated from an external perspective, the damage to the 'invisible' interior columns has, until know, remained a mystery. Given the amount of information that was available to us from the information unclassified sources, we conclude with the estimate that 7 to 20 core columns of the South Tower were destroyed upon impact.

Another interesting finding from this article stems from the consideration of safety factor and redundancy. The issue of symmetric versus asymmetric loading is an important one because unsymmetrical damage, i.e. the WTC towers, could be far more devastating in the global collapse scheme because the number of inactive columns is actually double the amount that were actually destroyed, while the amount of remaining, load carrying columns is reduced accordingly. Depending on the safety factor for which the towers were constructed, we have proven that the airplane impacts were able to bring the structures to the verge of collapse.

The prediction of the aircraft impact damage is summarized in table 3 showing the magnitude of energy dissipated by four major components involved in the collision, that is the airplane, exterior columns, floors, and core columns. Separate numbers are given for the North and South towers. The number of percentage of energy dissipated relative to the total available kinetic energy is given as well.

There were a number of factors that were not included in our analysis. For example the energy released through the explosion of jet fuel was not considered. Additionally, the effects of material and structural degradation as a result of the fires themselves were also not studied because these areas have been so extensively covered by others. Next, there has been no information on the average fragmented fuselage size, so there is no way to exactly determine the amount of fracture energy which was dissipated in the breakup of the aircraft itself. We did however, include the energy required to crush the fuselage, modeled as thin-walled cylinder and the energy to shatter wings. Finally, damage of exterior columns that were pre-stressed by the gravity load would have occurred in an explosive manner, sending around

large amplitude unloading waves that could additionally weaken the structure [34]. These and many other aspects of the accident reconstruction will be brought up in future analyses of the problem.

Table 3. Distribution of energy lost in the local damage of the TWC Towers
The energy is in the unit of MJ.

Energy (MJ)	North		South	
Airplane	586	23%	586	25%
Exterior	103	4%	122	3%
Floors	1221	48%	1925	53%
Core columns	630	25%	1025	28%
Total	2540	100%	3658	100%

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