Incorporating falling girder flexure into Nordenson's calculation

Claim:

When two girders impact each other and build up strain energy from the kinetic energy of the falling girder, then the equivalent force at maximum deflection can be calculated using a simple formula with the effective stiffness of both girders as springs in series. This is equivalent to calculating the strain energy for both girders separately, using each girder's individual stiffness and the condition that they both experience an equal but opposite force.

Proof

A subscript “$_b$” denotes the “girder **b**elow”, a subscript “$_f$” denotes the “**f**alling girder”

Formulas

(F1) Elastic strain energy is: $SE=\frac{1}{2}\cdot K\cdot D^2$

(F2) Deflection of an elastic element is: $D=\sqrt{(2\cdot SE/K)}$

(F3) Equivalent force is: $F=K\cdot D\Leftrightarrow F=K\cdot\sqrt{(2\cdot SE/K)}=\sqrt{(2\cdot SE\cdot K)}$

(F4) Formula for Effective Stiffness: $\frac{1}{K_{eff}}=\frac{1}{K_b}+\frac{1}{K_f}\Leftrightarrow K_{eff}=\frac{1}{1/K_b+1/K_f}=\frac{K_f}{K_f/K_b+1}=\frac{K_b\cdot K_f}{K_b+K_f}$

Algebra

When both girders build up strain energy concurrently up to the point where the falling girder comes to a stop, then

(1) $KE=SE_b+SE_f=\frac{1}{2}K_b\cdot D_b^2+\frac{1}{2}K_f\cdot D_f^2$

The forces acting on the girders are equal but opposite at all times:

(2) $F_b=-F_f\Leftrightarrow K_b\cdot D_b=-K_f\cdot D_f\Leftrightarrow D_f=-D_b\cdot K_b/K_f$

Now we can substitute D_f in (1) with the expression on the right of (2):

(1a) $KE=\frac{1}{2}K_b\cdot D_b^2+\frac{1}{2}K_f\cdot D_b^2\cdot K_b^2/K_f^2=\frac{1}{2}K_b\cdot D_b^2+\frac{1}{2}\cdot D_b^2\cdot K_b^2/K_f$

(1b) $KE=\frac{1}{2}D_b^2\cdot(K_b+K_b^2/K_f)=\frac{1}{2}D_b^2\cdot(\frac{K_b\cdot K_f}{K_f}+\frac{K_b^2}{K_f})=\frac{1}{2}D_b^2\cdot\frac{K_b\cdot K_f+K_b^2}{K_f}$

This can be solved for D_b:

(3) $D_b=\sqrt{2\cdot KE\cdot\frac{K_f}{K_b\cdot K_f+K_b^2}}$

The equivalent force to this is

(4) $$F=K_b\cdot D_b=K_b\cdot\sqrt{2\cdot KE\cdot\frac{K_f}{K_b\cdot K_f+K_b^2}}$$

(4a) $$F=\sqrt{2\cdot KE\cdot\frac{K_b^2\cdot K_f}{K_b\cdot K_f+K_b^2}}=\sqrt{2\cdot KE\cdot\frac{K_b\cdot K_f}{K_f+K_b}}$$

Comparing with (F4), we see that the division under the square root is the formula for effective stiffness of springs in sequence, K_{eff}.
This proves that treating the girders as springs in series is a correct way to calculate the equivalent force from available kinetic energy in the two-beam problem.

Example calculations

Values

The following values are fixed:

(V1) K_b = 7627 kip/in – from Nordenson's Table B5.1
(V2) KE = 3473 kip-in – from Nordenson's Table B6.1

Results

This table shows results for 5 cases – the stiffness value K_f for the falling girder is varied, the other values computed.

Case	K_f kip/in	K_{eff} kip/in	F kip	D_b in	D_f in	SE_b kip-in	SE_f kip-in	SE_b/KE
Cantilever (Szamboti)	3.7	3.6982...	160	0.0210	43.3	1.68	3471.32	0.05%
Cantilever K x 16	59.2	58.74	639	0.0838	10.8	26.7	3446.3	0.77%
1/16 of K_b	477	449	1766	0.232	3.70	204	3269	5.89%
Equal stiffness	7627	3814	5147	0.675	0.675	1736.5	1736.5	50%
Infinite K / Nordenson	∞	7627	7279	0.954	0	3473	0	100%

The second case happens to be close to Nordenson's shear failure value 632 kip for the connections to column 79 on floors 8-13 (Table B5.2)