WEB SITE: www.simulate-events.com

Technical Note No.64

Subject: **Large-deflection squashing of a wide-flange steel column**

This simulation was inspired by some controversy as to the mechanism of collapse of the WTC towers. The first orderly step in such an assessment is to determine the axial load capability of this type column in the damage zone. The deformation is intended to be large, taking the observer far past the peak load resistance the column can develop.

Figure 1 shows an over-all view of the column model used in this simulation. The constraining elements are 40 mm plates. The top constraints are deactivated. At the top end of the column only the vertical component of motion is allowed. The 'tributary mass', or the top ballast is equivalent to the portion of the building supported by the column. At the foot of the column only a horizontal movement is possible, as the axial component is restrained. The ends of the constraining members at the bottom can move vertically, but not in their plane.

The I or wide-flange section is rather thick so there is little natural tendency to local buckling.

The simulation of squashing is accomplished by a sudden application of a multiple of gravity load (step load). This causes the tributary mass to accelerate and apply a constant load to the column in a dynamic fashion.

The beginning of deformation after the peak resistance is attained is marked by formation of a plastic joint near the center of the column, as can be seen in Fig.2. Then, in Fig.3, the end plastic joints become visible. Marked cross-section distortion, especially at mid-height can be seen in Fig.4. Finally, in Fig.5 the column folds onto itself.

The resistance-deflection plot is provided in Fig. 6. The area under the line is the energy absorbed during deformation. The resistance force that the column provides varies strongly, therefore some simple measure of the energy-absorbing capability is desired. Let us denote the conventional buckling force by P_{cr}, the maximum deflection of the upper end by u and the average squashing resistance by ηP_{cr}. With this notation η is found by equating $u\eta P_{cr}$ to the area under the characteristic in Fig.5. Factor η is the fraction of the nominal strength retained by the column and is sometimes called the *retention factor*.

The resulting estimates of η are conservative, as they do not take one important factor into account. When the vertical deflection is as large as in Fig.5, the walls of a column are folding and leaning on one another. As the process continues and deflection progresses, it leads to quite large resisting forces being developed. The simulation presented here refrained from going into that range of deflection and taking advantage of this “end squash” effect, based on the reasoning that there is a limit to what the columns in a story below can support, even under dynamic conditions. Yet, in spite of it, the value of nearly $\eta = 0.587$ was attained. (With a conservative approach, using minimum rather than statistically probable yield strength , the value of η was reduced to 0.489.)

The conventional concept of buckling involves only P_{cr}. The first peak, shown on the right of Fig.5 has a considerably larger value due to reasons explained below.

DETAILS

The column material is A36 steel. Its nominal properties are

F_y = 36 ksi = 248 MPa (yield);
F_u = 60 ksi = 414 MPa (ultimate)
ε_u = 0.21 (ultimate strain)

The above steel was used to construct the WF-shaped columns, present in the core of the building, in WF shape. (In reality this was the weakest material used.)

The multiplying factors were used to allow for statistical distribution of strength. (The difference between the nominal or minimum guaranteed and the expected average properties). Employing the factor of 1.2 for F_y and 1.1 for F_u, the final values used were F_y = 298 MPa and F_u = 455 MPa.

The magnitude of gravity load applied suddenly to the ballast was such that the driving force was equal to $3P_{cr}$. (Sustained loading.) The benefit of the strain-rate effect was included by using the Cowper-Symonds coefficient of D = 0.0404 and q = 5, as appropriate for mild steel.

The section area, after it was simplified for this model, is 86,090 mm^2. The nominal yield load is P_y = 86,090 x 248 = 21.35 MN. After allowing for plasticity in buckling load calculations, we have P_{cr} = 21.0 MN, as the buckling load. The justification of the above near-equality is that the column is rather stocky, therefore the buckling load P_{cr} corresponds to a stress just below F_y.

The simulation was carried out during 180 ms, during which the column was almost completely squashed and shortened by u = 3,321 mm or 90% of its length. (The structural length was 3,680mm, as a bottom layer of elements, 46 mm thick, was added to the bottom end in Fig.1.) Further compression would lead to unreasonably high resistance values, not compatible with the strength of columns in a story below this one.

The integration result of the area under the curve in Fig.6 is 40.93×10^9 N-mm. When the equality mentioned at the outset is used, one has

$$\eta P_{cr} u = \eta\ 21.0 \times 10^6 \ \times 3{,}321 = 40.93 \times 10^9 \quad \text{or} \quad \eta = 0.587$$

One should note the difference between the peak resistance achieved by simulation, namely $P'_{cr} = 42.74 \times 10^6$ N = 42.74 MN and P_{cr} = 21.0 MN calculated above. There are three main reasons for this difference: Material strengthening due to strain-rate effect, lateral inertia, which stiffens the column against lateral buckling and the statistical effect mentioned above. The first two relate to physical behavior and little can be done about that. The last effect has some arbitrariness to it. In designing safe structures, the guaranteed minima are the right thing to use. However, for these steel types the tested yield strength is usually larger or much larger than the guaranteed values. If the objective is to find what happened, the most likely number must be used.

Still, the last step may look too bold for those not accustomed to the idea. To remove it from the picture, one observes that if F_y is returned to its original value of 248 MPa, the area in Fig.6 will also be reduced by the factor of 1.2 and η will change from 0.587 to η = 0.489.

How could a physical test differ from its virtual counterpart presented here? When marked bends form, there is a possibility of cracking at those locations. This is unlikely for a mild steel, like A36, but cracking at welded joints (top and bottom) can't be excluded. This factor could reduce the column resistance. Also, a significant eccentricity will degrade the column post-buckling strength.

One should also note a certain degradation of the results because of numerical reasons. At about the mid-path on the abscissa in Fig.6 the plot becomes jagged. This is due to erosion (loss) of elements caused by the user settings during the run. It was necessary to eliminate elements showing excessive distortion, but such an elimination has somewhat decreased the axial capacity of the column.

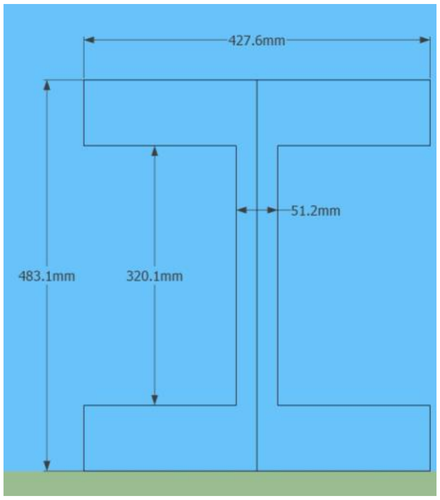

Fig.1a Simplified section of the column

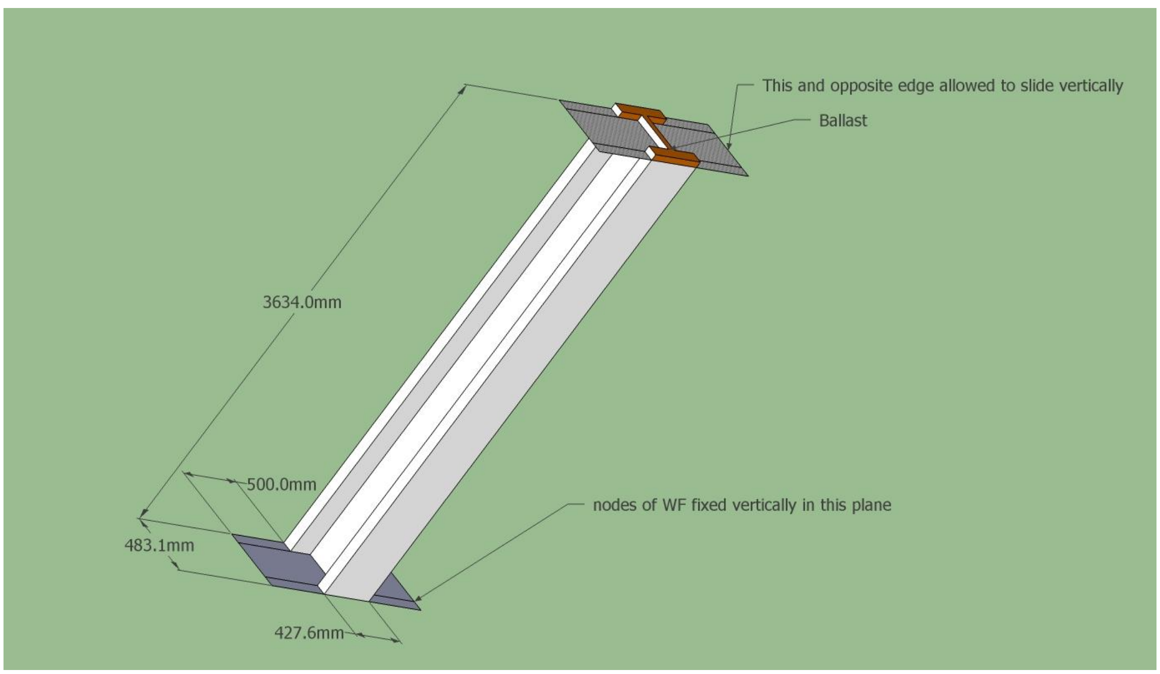

Fig.1b Axonometric view of the column with end restraints

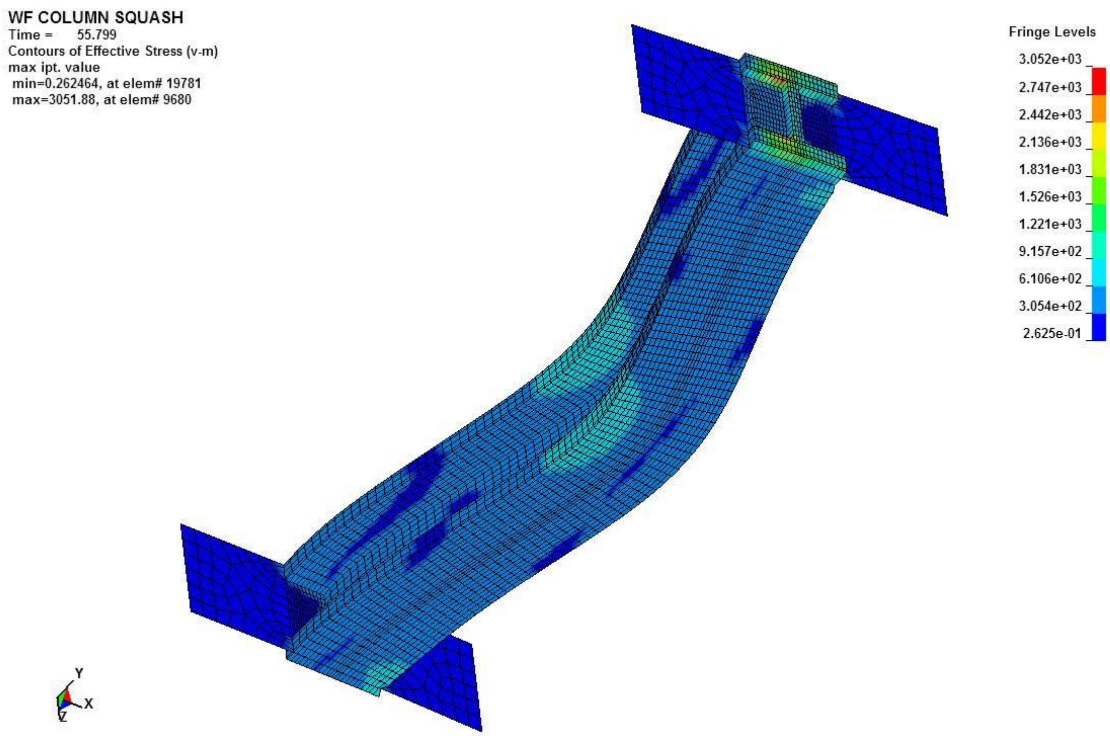

Fig.2 Deflected shape after 56 ms. A plastic joint begins to form about mid-height.

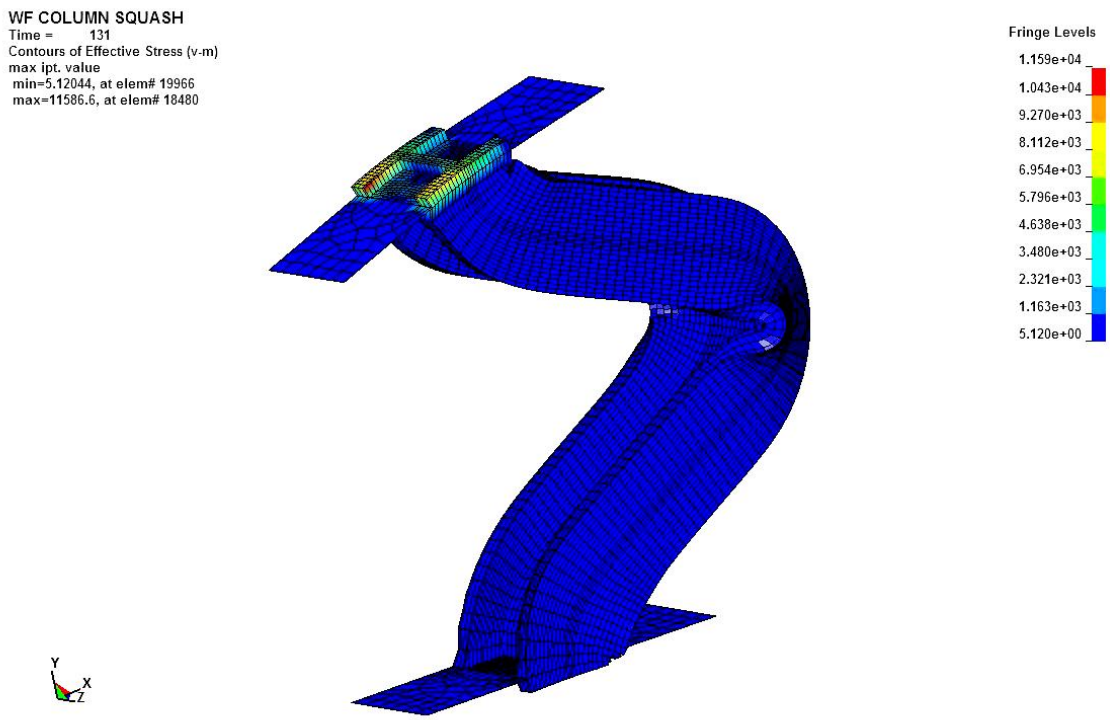

Fig.3 Deflected shape after 131 ms. The end plastic joints also become visible.

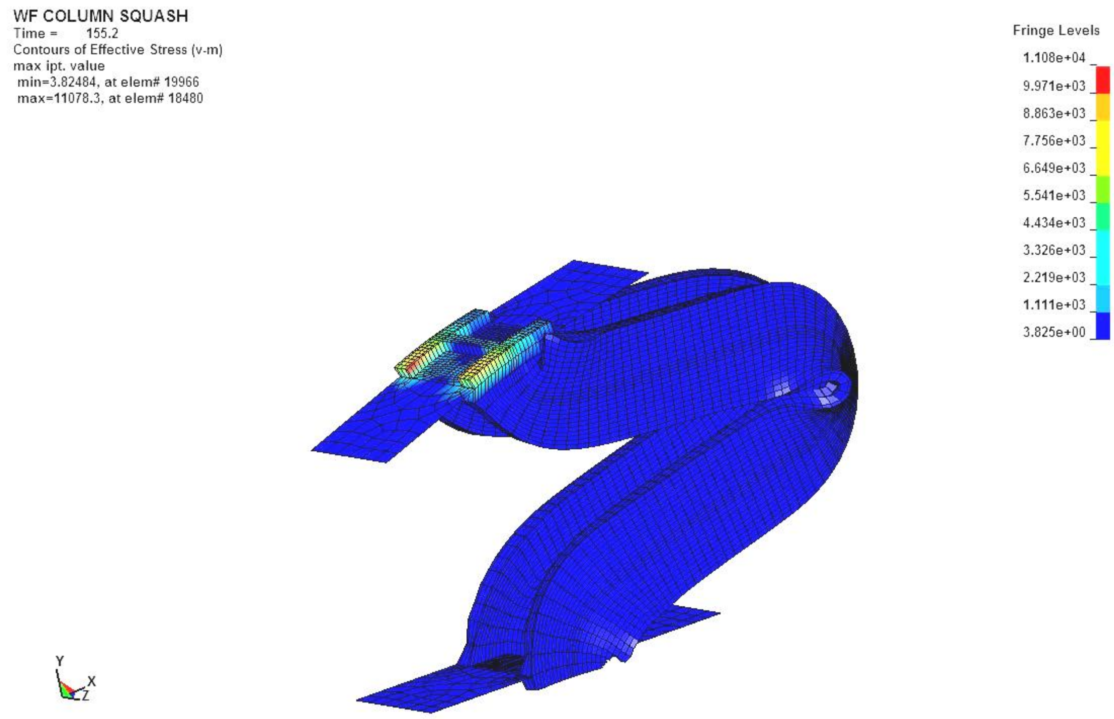

Fig.4 Deflected shape after 155 ms. The plastic joints become extreme.

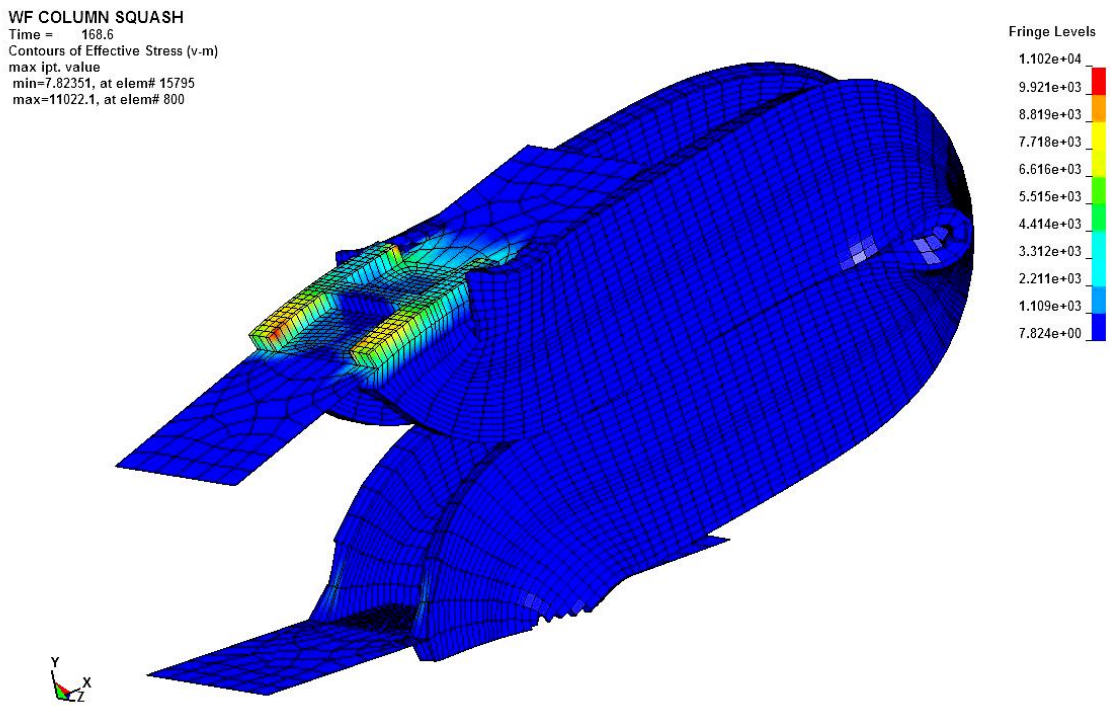

Fig.5 Deflected shape after 169 ms. Squashing is practically complete.

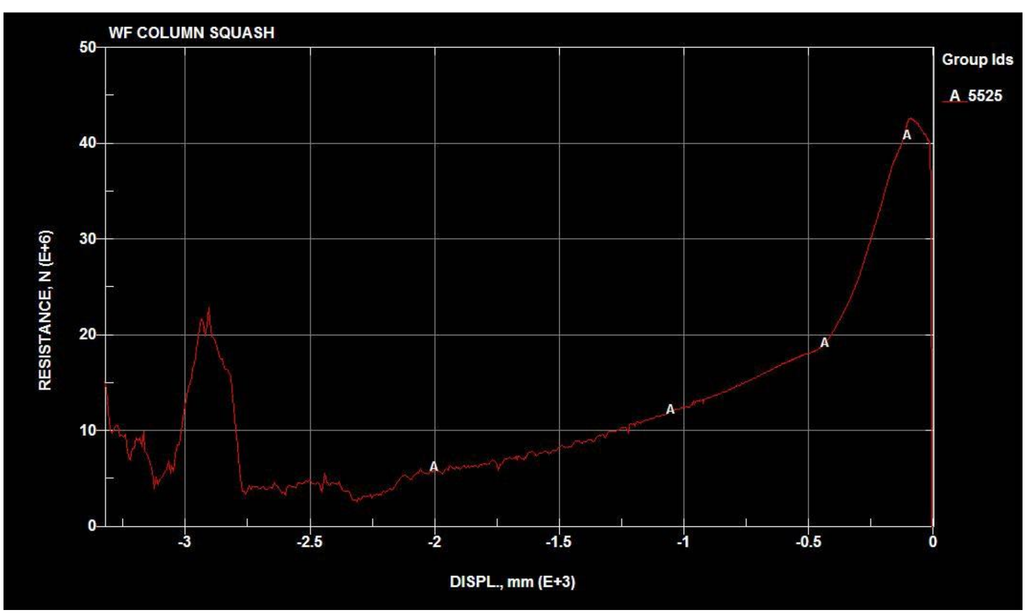

Fig.6 Resistance (meganewtons) versus top displacement, meters.